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Report of project group 1 on

Using oil booms to clean up large oil spills

Clean up oil spills by using Oil-booms

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1 General problem description

In case of an oil spill at sea, stopping the spill from reaching shore is critical. One of the most important tools in such operations is the oil containment boom, floating booms that stop the formation of large oil films, concentrating the oil in thicker layers that can be skimmed. The Norwegian company Norlense produces containment booms of different types for different operating conditions and performs full scale tests with this equipment. The assignment will consist in modeling one or more aspects of oil boom operation, and possibly to investigate some proposed modifications Norlense are working on. We try to develop some techniques which can efficiently clean up the floating oil which is caused by leaking from boat or industrial failure. The difficulties are caused by natural turbulence on the sea surface and mix-up effect of oil and water.

We mainly use two approaches:

- Floating Platform
- Vortex Oil Boom

2 Approach 1: Floating Platform

2.1 Problem Description

As the main difficulty of collecting oil spill on sea surface is caused by wave and turbulence in seawater, our first approach focuses on calming down water turbulence, so that oil droplets could float and would be easier and more efficient to collect. The overall idea is moving a floating rectangle open channel on sea surface with certain velocity, such that when seawater enters the platform, the turbulence would claim down within certain period of time which depends on Reynolds number. And further device would be right after the platform to collect (floated oil droplets) separated oil from the water. In this case we need to design an optimal size of the platform, which is including the length, width and depth. Also we have to calculate the velocity of dragging platform so that oil droplets have enough time to float up to the surface. The situation is illustrated in the figure below.

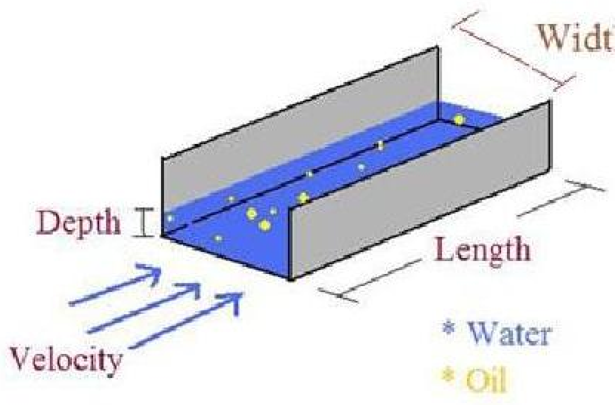


Figure 1: platform

2.2 Mathematical Model

In the procedure of determining the above quantities, we have used the following mathematical theories and physical laws.

1. **Stokes' Law** Stokes' law for drag force is expressed as:

   $$F_d = 6\pi\mu V d,$$

   where:

   - F_d is the drag force of the fluid on a sphere
   - μ is the fluid's dynamic viscosity
   - d is the diameter of the sphere.
   - V velocity of the sphere relative to the fluid

2. **Newton's Law**

   $$F_{net} = ma,$$

   where:

   - F_{net} is the force acting on an object
   - m is the mass
   - a is the acceleration of the object

3. **Archimedes' principle** "Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.".

   $$F_{up} = \rho V g,$$

   where:

   - F_{up} is the buoyancy
   - ρ is the density of fluid
   - V is the volume of object in fluid
   - g is the gravitational acceleration

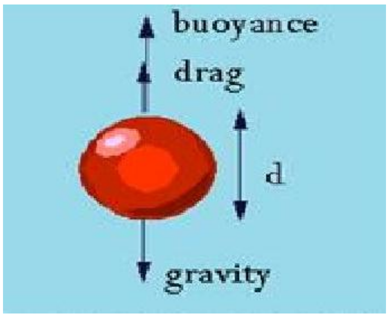
4. **Reynolds number** The Reynolds Number Re describes whether flow conditions around a sphere are laminar or turbulent.

$$Re = \frac{\rho V D}{\mu},$$

where:

- ρ is the density of fluid
- V is the fluids velocity
- μ is the dynamic viscosity of the fluid
- $D = \frac{4dw}{2d+w}$ (d - depth, w - width)
hydraulic radius which is the cross-sectional area of the channel divided by the wetted perimeter.

Now, let's come back to our case. We consider the following free body diagram.



The net force F_{net} acting on the oil droplet is given by

$$F_{net} = F_d + F_u - mg.$$

Then Newton's second law implies $F_d + F_u - mg = ma$, which also implies $6\pi\mu V d + \rho_w V_0 g - mg = ma$ where V_o is the volume of the oil droplet.

After replacing m by $\rho_o V_o$, V_o by $\frac{4}{3}\pi R^3$, V by $\frac{dx}{dt}$, a by $\frac{d^2x}{dt^2}$ and then rearranging , we get the following ordinary differential equation

$$\frac{d^2x}{dt^2} + \frac{9}{2}\frac{\mu}{\rho_o R^2}\frac{dx}{dt} - g(\frac{\rho_w}{\rho_o} - 1) = 0$$

We assume the following initial conditions when the platform stabilizes the seaconditions:
$x(0) = -5$ ($5m$ below oceansurface, i.e., position of the deepest oil droplet)
$x'(0) = 0$, oil droplet does not move up or down

These two conditions combined with the radius of the smallest oil droplet $R = 10^{-3}m$ can be thought as the worst case scenario. Other droplets are bigger and closer to the surface and it is easy to see that they rise up faster.

We consider other constants that represent seawater and oil:

Viscosity of water $\mu = 0.001kg/(ms)$

Density of water $\rho_w = 1100kg/m^3$ (seawater is denser than freshwater)

Density of oil $\rho_0 = 900kg/m^3$ (example for automobile-oil: petroleum diesel)

Gravity $g = 9.81m/s^2$

2.3 Numerical Results

Assumption: We assume laminar flow after the platform is put in to the ocean: Solving the ODE under the initial conditions, we get the time taken for the smallest and deepest (which is actually the worst case) oil droplet to float to be 11.66 sec.

Under this assumption, by fixing one of the variables V or L, we get limitation for the other. For example, if L is fixed to $20m$, then V has to be slower than $1.7\frac{m}{s}$ to allow the droplet resurface within the box in case of laminar flow.

Also analyzing the Reynolds number we see that we cannot say that the flow would be laminar in any usable speed of towing and reasonable dimensions of the platform.
For example with depth being 5 meters, towing speed $0.1\frac{m}{s}$, and width of the platform just 20 cm, Reynolds number would be over 40000.

With these non laminar flows we do not know how fast droplets would resurface. It is to be expected that the resurfacing would be slower than in completely laminar flow. And additionally there is no indication of how fast this stabilization would occur. This means that some additional length has to be added for the platform to give the flow time to stabilize and then allow the resurfacing happen in this stabilized flow.

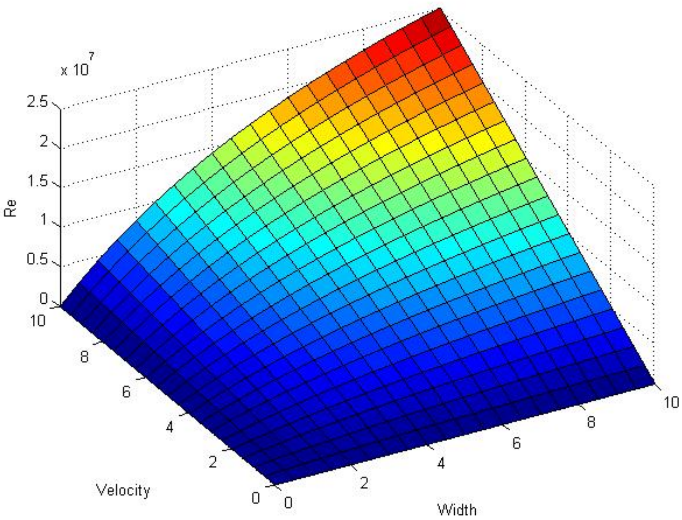


Figure 2: Reynolds number

2.4 Conclusion

It is very difficult to draw conclusion so far due to the following limitations.

- We considered only the laminar flow
- We did not calculate how long time is required for the transition of the flow form turbulent to laminar. We are not also sure if this transition really occurs under our problem
- We did not consider some factors such as temperature, wind etc.

3 Approach 2: Vortex Oil Boom

3.1 Problem Description

The approach is using a moving asymmetric boom to create a vortex in the sea, and by centrifugal force, to collect spilled oil. One essential part of the problem is to determine the behavior of the oildroplets in the vortex. This information can then be used while determining the shape of the boom and the efficiency or problems of the approach. Therefore we used the following theories and formulas:

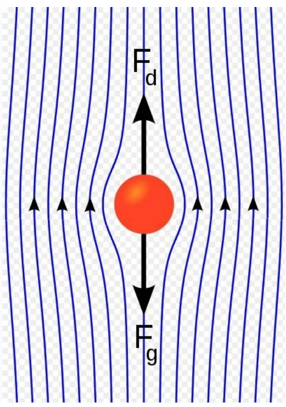
3.2 Mathematical Model

a) Stokes' Law

$$F_d = 6\pi\mu RV,$$

where:

- F_d is the frictional force
- μ is the fluid's dynamic viscosity
- R is the radius of the spherical object
- V is the particle's velocity
- F_g is the gravity force



b) Centrifugal Force

$$F_c = M\frac{v^2}{r}$$

where:

- M is the mass
- v is the velocity
- r is the radius

c) Vortex motion model

$$
\begin{aligned}
\frac{\partial^2\Psi}{\partial x^2} + r\frac{\partial\Psi}{r\partial r} + r^2F'(\Psi) + \Phi(\Psi)\Phi'(\Psi) &= 0 \\
V_x = - & \tfrac{\partial\Psi}{r\partial r} \quad \text{velocity upwards} \\
V_r = & \tfrac{\partial\Psi}{r\partial x} \quad \text{radial velocity} \\
V_\varphi = & \tfrac{\Phi(\Psi)}{r} \quad \text{angular velocity}
\end{aligned}
$$

where:

- Ψ is the stream function
- $\Phi(\Psi) = k\Psi$
- energy distribution law:

$$F(\Psi) = \frac{1}{2}\epsilon^2\Psi^2 + C$$

- ϵ, C and k are constants.

From this we get the connection to rotation and suction.

$$V_x = -\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{V_\varphi r}{k}\right)$$

We did not use correct stream function, but instead approximated that by rotating disk, which is rather rough but will suffice for this purpose since the actual vortex obviously not like this anyway. So in the end what we have is just

$$V_x = -\frac{C_2}{rk} \tag{1}$$

where C_2 is just angular speed of our rotating disk.

We actually cut the downwardstream to constant from some radius to not to have these infinite speeds:

$$V_x = \begin{cases} -\frac{C_2}{rk}, & \text{if } r > C_3 \\ -\frac{C_2}{C_3k}, & \text{if } r < C_3 \end{cases} \tag{2}$$

3.3 Model for radial movement

For simplification, we consider the system as a rotating disk, and combine it with vortex motion model which defines the connection of the rotation and the vertical motion.

For the radial motion of a droplet we get system:

$$r'' + \frac{9}{2}\frac{\mu}{\rho_o R^2} r' - (1 - \frac{\rho_w}{\rho_o}) r \quad = \quad 0$$

By solving this ODE, we get the motion of the oil droplet towards the center of the rotation, which you can see in Figure 3. This system corresponds to the case of rotating disk and so if that corrected by something more reasonable this system will also change. The change will occur in the term caused by the centrifugal force, that now corresponds to water circling around. If suction occurs this force would actually be stronger in order to put the water into a spiral motion as well.

3.4 Model for downward motion

This is the first case combined with downward stream according to position in the vortex as in (2). For the vertical motion we use the approach from the first case and combine it with the vortex motion model. By combining vertical and radial motion we have a model for the motion of the oil droplet in vortex (see Figure 4).

4 Improvements for the first case

We got an idea that if this rotational flow could be created in the platform, without the downward suction because the bottom is closed, the droplet resurfacing phenomenon would speed up due to the concentration of the oil in water. So basically just combining first and second case ideas of the company to produce something better. And now the size of the vortex or rotational flow would be small and maybe existing in real world.

Configuration as in figure 7 would not work actually because not all water goes thru these rotations but some better configuration should be considered. This would be shape optimization of a platform with flow. Easier than the 3-d vortex but still a quite difficult. In any case the idea should be taken to consideration because the only relevant thing was to speed up the separation

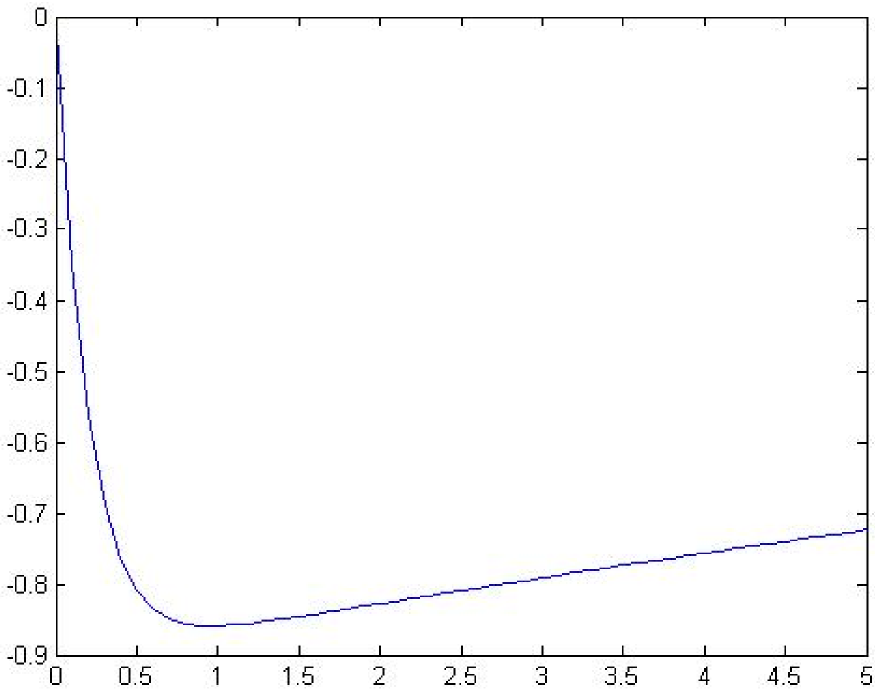


Figure 3: Speed of $10^{-3}m$ droplet out from the center with initial position r=20m

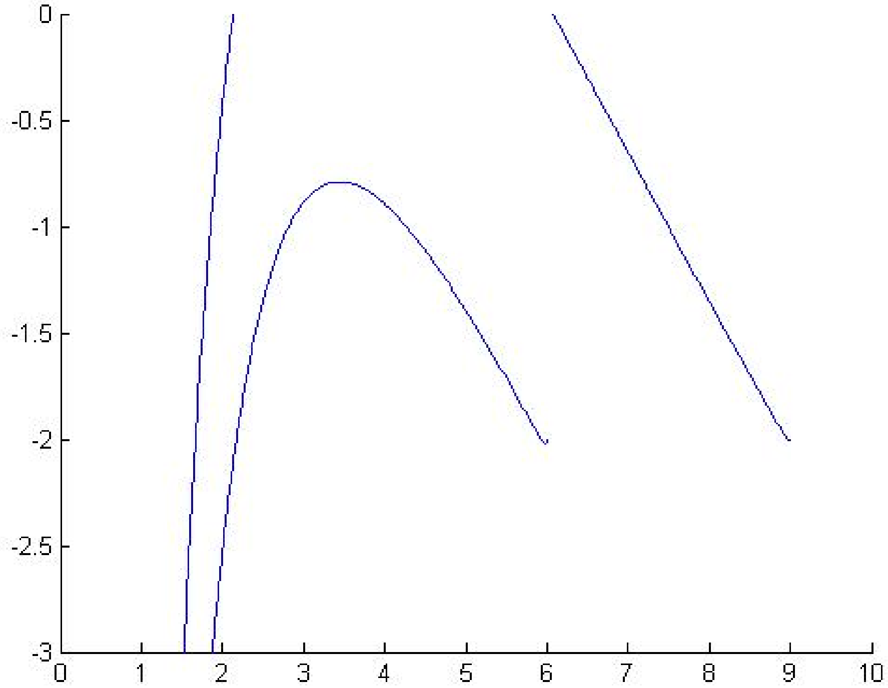


Figure 4: Oil droplet paths in a vortex

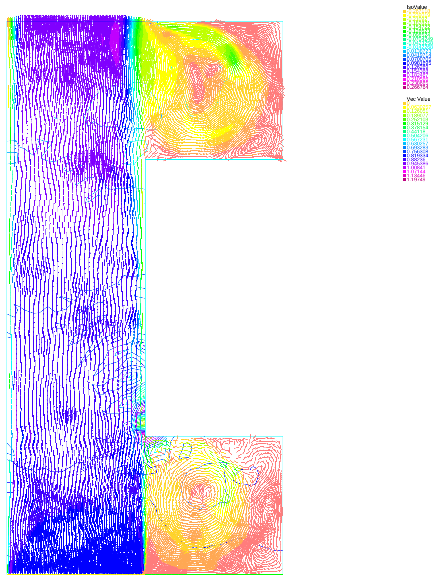


Figure 5: Flow in channel with some deformations

of oil from water and this is exact what this would do. And also here the mathematical model could actually be good enough so that the solution of the optimization could be somewhat realistic. And now our model for the second case would tell something of movement of the droplets, so it could actually be used in the optimization process.

5 Improvements for the second case

Since we do not know the exact shape or strength of the vortex, and since this is essential in determining the effectiveness of the procedure, we instead suggest an improvement that would allow to control the strength of the vortex independent of the pulling speed. This with the analysis that we did for the behavior of oildroplets in the vortex allows the search of the optimum angle of the additional boom line for the oil-water separation.

It must be mentioned that these two computations include an assumption that the suction would be created. The comparison of these two shows that the additional boomline would direct the flow in such a manner that a stronger rotational flow would be created with same pulling speed. And again according to our model the rotational speed of the water in the vortex is key defining factor in improving the separation of oil and water.

Even if it is hypothetical at this point whether the suction effect occurs or not it is important to understand that the rotation of the water most

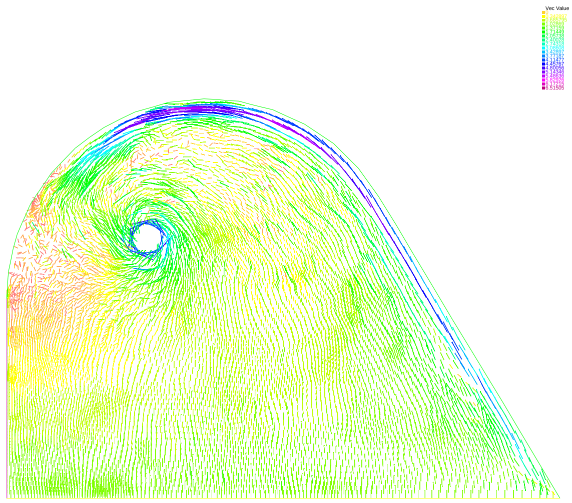


Figure 6: Oilboom vortex with regular setup

likely will. So even the water would escape completely under the boomline and not thru the vortex, the rotation of the water exists. And would in that case actually resemble more of this rotating disk of ours, but without the suction effect that in our model was a weakening factor of the procedure.

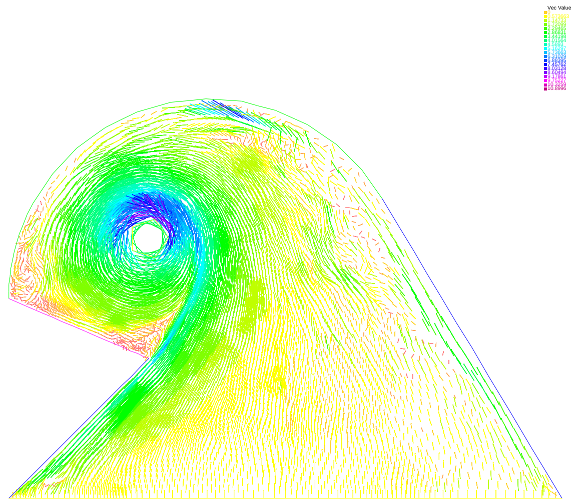


Figure 7: Improved oilboom with additional boomline

The computations are done with FreeFem++ with discretization of the velocity pressure formulation and using FreeFem++:s convection operator to give discretization in time [5]. For these computations boundary conditions were chosen corresponding to

$$\begin{aligned}&\mathbf{u} = 0 \text{ on the oilboom,}\\ &\mathbf{u} \cdot n = g \text{ on the open boundary}\\ &\textstyle\int_\Gamma \mathbf{u} \cdot n = 0\end{aligned} \tag{3}$$

where Γ is the union of the inner circle representing the suction of water into the vortex and the open boundary from where the water comes in to the domain as the oilboom is dragged after the ships. And g is the velocity at which the oilboom is pulled, so the velocity at which the water comes into the computational domain. The existence of the vortex with suction is assumed and approximated by the inner circle in the computational domain into where the water will vanish from this 2-D flow. To conclude we suggest this additional boomline in order to improve the strength of the vortex. This includes the idea of being able to control this line into wanted angle in order to prevent too strong suction which would, according to our model, weaken the resurfacing of oil droplets.

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