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Report of project group 6 on

Optimal strategies in the electricity market

Optimal Strategies on the Electricity Market

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Abstract

This paper on financial optimization is the result of a project carried out during the European Consortium for Mathematics in Industry's Modelling Week in Wroclaw, Poland 2009.

During the last two decades the Scandinavian electricity market has been progressively deregulated: Whereas 20 years ago governmental institutions were handling the distribution of electricity, this is now managed by private companies who buy and sell electricity at the Nord Pool Exchange. The exchange price is highly volatile, and market participants are exposed to a significant price risk.

In this project we present and model strategies of how an electricity retailer company can optimize profit while minimizing risk when allowed to make daily future contracts.

1 Introduction

In the followings of the deregulation of the energy markets in Norway and Sweden in the early 90's, the first multinational energy exchange, Nord Pool, was created in 1996 as a market place for both Swedish and Norwegian trading of energy [1]. Since then it has grown to include the energy markets of Finland, Denmark and northern parts of Germany [2].

As on any other exchange participants use mathematical methods to minimize the price risk while maximizing gain. However, unlike most other commodities, electricity cannot be stored. This affects some earlier established theories [3]. We have tried to model optimal trading portfolios for electricity retailer companies dealing on the Nord Pool Exchange.

1.1 Introductory finance

As this paper is written in context of ECMI's Modelling Week in Wroclaw 2009, we present to readers not familiar with the subject a few facts on finance.

An exchange is a market where buyers and sellers of some asset or financial contract meet to operate. Everyone deals with everyone. You simply decide if you would like to buy or sell for the market price, known formally as the spot price. The spot price is set by the laws of supply and demand. If more participants buy than sell, the price goes up, and vice verca. Nord Pool is an exchange market where electricity is the main asset.

A derivative is a financial instrument whose price derives from some other underlying variable. Most typically this will be the price of an asset, but it could also be other variables such as weather, expectations of the market etc. Two commonly used derivatives are futures and options. An electricity future contract is an agreement between two parties to buy or sell electricity for a certain price at a certain date. There is no cost for entering into a futures contract. The price is called the future-price, and the execution date is known as the maturity of the contract. At an exchange the future price for futures with different maturities could depend on many different variables, for example the expectations on the development of the spot price [4], [3]. An electricity *option* gives the holder the option to buy/sell electricity for a certain price at a certain date. The price for which the holder can buy/sell electricity is the strike price, and option price is the price the holder pays to enter into a contract.

Entering into future and option contracts in order to lower ones exposure to risk is known as *hedging.*

2 A simplified model

We are seeking an optimal strategy for a company dealing on the Nord Pool Exchange. That is, we wish to find a way of buying assets and contracts such that expected gain is maximized given a certain risk, *or* risk is minimized given a certain expected gain. Due to the complexity of this task, we make certain simplifications. First, we eliminate

Symbol	Description
S_t	Spot price on day t
D	Demand
F_t^T	Forward price at time t for a contract with execution day T
f_t^T	Size of corresponding forward contract
K	Price for consumers
X	Profit on day T

Table 1: Model parameters and variables

option contracts, and only consider futures. It is possible to make daily, monthly, quarterly or yearly future contracts. However, we restrict our attention to daily contracts. Furthermore, we consider the daily demand of our customers to be known and constant. Table 1 lists the model variables and parameters.

To summarize, we find ourselves in the following situation: On day t we wish to determine how to meet the demand of electricity D on day T. Should we buy a future contract for the entire demand D today? Should we take the risk of waiting, and buy everything at the spot price on day T? Or is there perhaps some optimal day between t and T on which we should enter into a future contract? Three different approaches suggest answers to these questions, and are presented in Sections 3, 4, and 5.

3 Forecasting spot prices

As the spot price is highly volatile, it is challenging to forecast its behavior. A long-term forecast (i.e. 6 months forward) is virtually impossible to produce, as it can change drastically for no apparent reason. However, a short-term forecast with a decent level of accuracy can be constructed using simple averaging methods. We will take a look at two slighty different methods for deriving an estimate for the spot price tomorrow.

3.1 Weighted moving average

A weighted moving average (WMA) gives different weights to different data points. The general formula for a weighted moving average is

$$\hat{X}_{t+1} = \frac{\sum_{i=0}^{n} \omega_i X_{t-i}}{\sum_{i=0}^{n} \omega_i}.$$

We assume that including the spot prices of today and six previous days is enough to produce a decent forecast for tomorrow ($n = 6$). Most recent spot prices, such as that of today, are given the most weight. More specifically, our WMA estimate of tomorrow's spot price is given by

$$\hat{S}_{t+1} = \frac{1}{28} \sum_{i=1}^{7} i \cdot X_{t-7+i}.$$

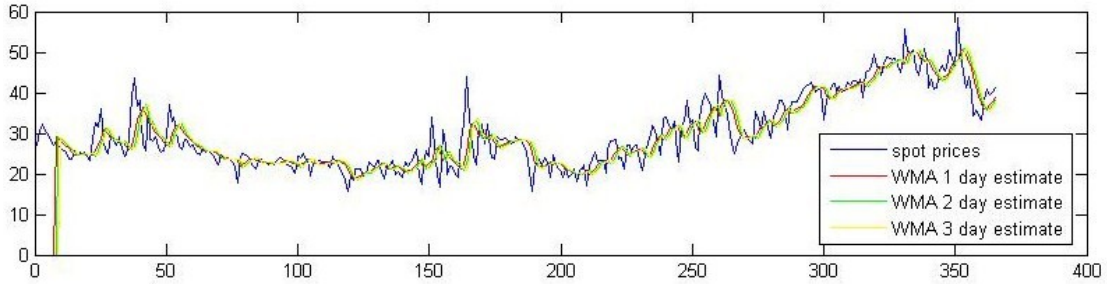

Figure 1: Spot prices and corresponding WMA estimates. Used data is Nord Pool spot prices in Helsinki 2007.

Taking this further, we could make estimates of the spot price two or three days ahead by assuming $\hat{S}_{t+1} = S_{t+1}$ and $\hat{S}_{t+2} = S_{t+2}$, yielding the formulas

$$\hat{S}_{t+2} = \frac{1}{28}\left(\sum_{i=1}^{6} i \cdot X_{t-6+i} + 7\hat{S}_{t+1}\right)$$

and

$$\hat{S}_{t+3} = \frac{1}{28}\left(\sum_{i=1}^{5} i \cdot X_{t-5+i} + 6\hat{S}_{t+1} + 7\hat{S}_{t+2}\right).$$

Figure 1 shows these three estimates, and the true spot price, plotted over a year.

3.2 Exponentially weighted moving average

Alternatively, an exponentially weighted moving average (EWMA) can be used to estimate tomorrow's spot price. For an EMWA, weights reduce exponentially with time, and the general formula is

$$\hat{X}_{t+1} = \alpha\left(X_t + \sum_{i=1}^{n} (1-\alpha)^i X_{t-i}\right), \quad 0 \leq \alpha \leq 1,$$

where α is a constant smoothing factor. For our data set $\alpha = 0.7$ seems to be a good fit, and again we assume that averaging over the previous week (n=6) suffices for a decent approximation. The resulting estimate of tomorrow's spot price is given by

$$\hat{S}_{t+1} = 0.7\left(S_t + \sum_{i=1}^{n} (1-0.7)^i S_{t-i}\right).$$

Similarly to what was done in section 3.1, we can extend this to approximate spot prices two or three days ahead by formulae

$$\hat{S}_{t+2} = 0.7\left(\hat{S}_{t+1} + \sum_{i=1}^{n} (1-0.7)^i S_{t+1-i}\right)$$

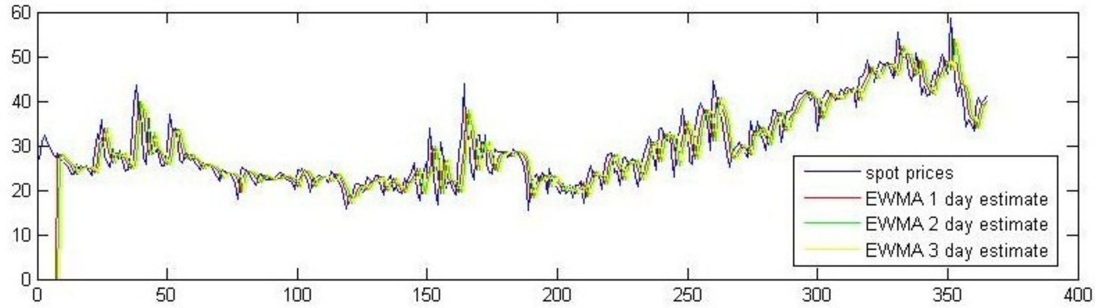

Figure 2: Spot prices and EWMA estimates. Used data is Nord Pool spot prices in Helsinki 2007.

and

$$\hat{S}_{t+3} = 0.7\left(\hat{S}_{t+2} + (1-0.7)\hat{S}_{t+1} + \sum_{i=2}^{n}(1-0.7)^i S_{t+2-i}\right).$$

Figure 2 shows these estimates, and the true spot price, plotted over a year.

3.3 "Last minute hedging"

Say we find ourselves in the following situation: We know our customers will require D units of electricity tomorrow, and we have not yet made a future contract to meet this demand. This means that we must either enter into a future contract today to cover the remaining demand, or we will have to buy the remaining units of electricity at the spot price tomorrow. From the estimates found in Sections 3.1 and 3.2, we find a simple decision rule to apply in this situation: If $\hat{S}_{t+1} \geq F_t^{t+1}$, we make a future contract for the remaining demand of tomorrow today. If $\hat{S}_{t+1} < F_t^{t+1}$, we wait and buy at spot price. As Nord Pool is closed on weekends, the decision of what to do for Sunday and Monday is made on Friday, comparing F_t^{t+2} with $\hat{S}_{t+2}$ and F_t^{t+3} with $\hat{S}_{t+3}$, respectively.

Figure 3 shows the cumulative profit over a year (for a company in Helsinki, 2007) using three different stategies; 1) always making future contracts, 2) always buying at spot price, or 3) using the given decision rule. We see that this "last minute hedging" allows us to make close to maximal profit (that of always buying at the spot price), with the additional benefit of sometimes knowing beforehand how the demand of tomorrow will be met.

Our decision rule of how to meet tomorrow's electricity demand seems to work well. The EMWA method gives slightly better estimates of the spot price than the MWA method, particularly at times when the spot price is most volatile. However, whichever method we use, we are not lowering a company's risk significantly. Such short-sighted forecastings are unable to incorporate the seasonality components that characterize the behaviour of the spot price and, moreover, are subject to gross errors during periods when the market volatility is higher than usual. In order to truly obtain a reduced risk, we must look further into the future.

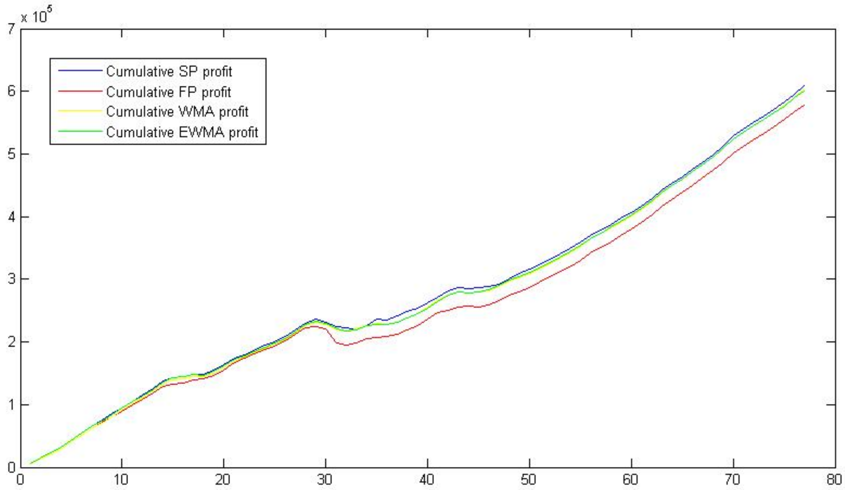

Figure 3: Cumulative profits using three different strategies: Buying everything at spot price (SP), always making a future contract (FP), and the "last minute hedging" strategy, calculated using either WMA or EWMA estimates for the spot price.

4 Static model: Scheduling from Monday to Saturday

Now consider the following task: Given a certain risk threshhold, our electricity retailer company asks for the optimal strategy over the week in order to maximize income on Saturday. That is, starting on Monday, we must determine how many future contracts for Saturday to buy or sell on each day of the week. As a simplification, we say that this strategy is nailed on Monday, and is not changed depending on developments in the market through the week.

Our income on Saturday will be given by the formula

$$X = (K - S_T)D + \sum_{i=1}^{5} (S_T - F_i^T) f_i^T .$$

If we decide to measure the risk of our position as the standard deviation of our income, it is now possible to model our problem as

$$\max \mathbb{E}(X) \text{ subject to } StDev(X) \leq \alpha.$$

With a fixed demand D, the expressions for $\mathbb{E}(X)$ and $StDev(X)$ are given by

$$\mathbb{E}(X) = (K - \mathbb{E}(S_T))D + (\mathbb{E}(S_T) - F_1^T)f_1^T + \sum_{i=2}^{5} (\mathbb{E}(S_T) - \mathbb{E}(F_i^T))f_i^T \ ,$$

$$StDev(X)^2 = (\sum_{i=1}^{5} f_i^T - D)^5 Var(S_T) + \sum_{i=2}^{5}\sum_{j=2}^{4} f_i^T f_j^T Cov(F_i^T, F_j^T)$$
$$+ 2\sum_{i=2}^{5} Cov(S_T, F_i^T) f_i^T (\sum_{j=1}^{5} f_j^T - D) \ .$$

Thus, we need to find reliable estimates for the expected values, variances and covariances in these formulae.

How do we find such estimates? From qualitative observations on the historical data, we detect seasonal components in the behaviour of the spot and future prices. In particular, we observe strong daily seasonality (electricity is cheaper during night and the price increases during working hours) and weekly seasonality (the price decreases over the weekend). The yearly seasonality is less evident and often confounded by other real world events that influence the price of energy (e.g. wars, catastrophes, economical crisis). Since we are scheduling over a single week we ignore the yearly and daily components, while the weekly seasonality is accounted for. Furthermore, we observe that in general, the price of the future contracts is almost constant at the beginning of the week and shows fluctuations only on the 2-3 days before their execution date. According to these observations we decide to use the following estimates

$$\widehat{\mathbb{E}(S_T)} = mean\{S_i : i = Saturday\},$$
$$\widehat{\mathbb{E}(F_i^{Sat})} = F_{Mon}^{Sat} + \mathbb{E}(F_i^{Sat} - F_{Mon}^{Sat}) = F_{Mon}^{Sat} + mean\{F_i^{Sat} - F_{Mon}^{Sat}\} \ ,$$

and we compute the sample covariance matrix for the data sets $X_i^j = (F_i^{Sat})^j - (F_{Mon}^{Sat})^j, i = 1, ..., 5$, and $X_6^j = \{S_j : j = Sat\}$, using the maximum-likelihood estimators

$$\Sigma = 1/(n-1)\sum_j (X^j - \bar{X})(X^j - \bar{X})^T,$$

where $\bar{X}$ is the mean vector $[mean(X_1), ..., mean(X_6)]$.

This model is analysed using spot price data from 2005, where all prices are given in Norwegian Kroner (NOK). The demand on Saturday is set at 300 MWh, and our customers are paying 350 NOK/MWh. We assume that future contracts are bought and sold in clusters of 50 units, and we only allow ourselves to sell future contracts bought in the same week. Under these assumptions, we are able to compute the expected gain and corresponding standard deviation for all strategies we might adopt. The results are shown in Figure 4. We can now set a risk level that our company considers acceptable, and for any such level determine a corresponding optimal strategy (i.e. we select the strategy that yelds the greatest expected gain among those whose risk is lower than our

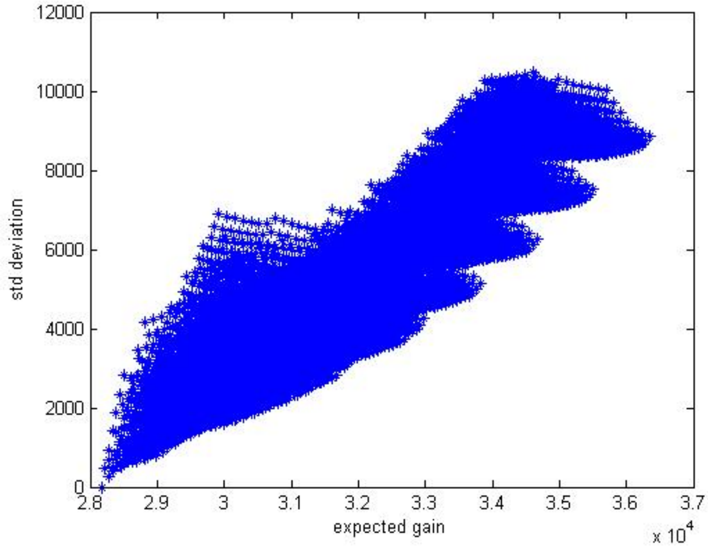

Figure 4: Expected gain and standard deviation for a set of different strategies.

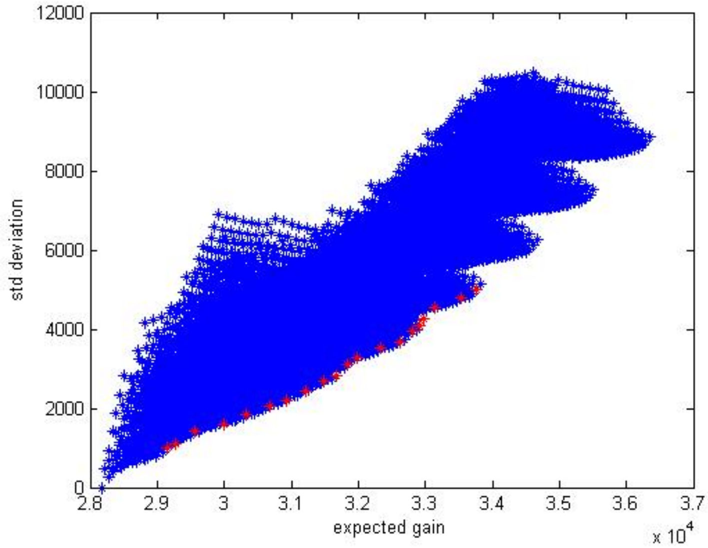

Figure 5: Optimal strategies, marked red, for 20 different risk levels.

α level	$\mathbb{E}(X)$	Mon	Tue	Wed	Thu	Fri	X
1000	29150	150	0	50	50	50	30758
1842	30338	150	0	0	50	50	30275
2895	31665	0	0	0	100	150	32442
3947	32810	0	150	-150	0	200	30972
5000	33766	0	50	-50	0	150	31189

Table 2: Optimal strategies and true gains for 5 different risk levels

$X > \mathbb{E}(X)$	$\mathbb{E}(X) > X > \mathbb{E}(X) - \sigma$	$\mathbb{E}(X) - \sigma > X > \mathbb{E}(X) - 2\sigma$	$X < \mathbb{E}(X) - 2\sigma$
0.36	0.35	0.17	0.12

Table 3: Results of backtesting on the data from year 2005, $\sigma = StDev(X)$.

risk level threshold). A graphical representation of chosen strategies is shown in Figure 5. Table 2 displays optimal strategies for a few specific risk levels.

Table 2 shows that the calculated expected gains are quite consistent with the corresponding true gains when using the selected market strategies. In order to check the reliability of this method, we perform more backtesting: When computing optimal strategies for 20 different risk levels for 18 weeks in the second half of 2005, and then comparing expected and true gains using these strategies, we get the results presented in Table 3. The results are quite encouraging: The actual gain is greater than our expectation in 36% of all cases, while it is within one standard deviation in 35% of the cases. In 17% of all cases the actual gain is between one and two standard deviations lower than expected, and in only 12% of all cases is the true gain more than two standard deviation lower than expected. Overall we find these to be decent results. It should be fairly easy to further enhance them using better estimates for the spot and future prices, and by introducing mechanisms that let the strategy change in the following days according to the new market situation.

5 Convex optimization approach

In this section, we present an enhancement of the model presented in Section 4. We still only consider daily future contracts, and we let the demand of our customers D remain constant. Let t be the date from which we are planning our strategy (say today) and n be the date for which we are buying electricity. This means we have $n - t$ days with the option of buying future contracts for day n. Our goal is to find a buying strategy $\xi_n = (f_t^n, f_{t+1}^n, \cdots, f_{n-1}^n, s_n)$, where s_n is the amount of energy we buy at the spot price on day n.

The demand D must be satisfied, hence we have to require $\sum_{i=t}^{n-1} f_i^n + s_n \geq D$. The expected price we will have to pay for this amount of electricity is given by

$$\mathbb{E}[C^n] = f_t^n F_t^n + \sum_{i=t+1}^{n-1} f_i^n \mathbb{E}[F_i^n] + s_n \mathbb{E}[S_n] \ ,$$

and we seek to minimize this $\mathbb{E}[C^n]$. Clearly, $\mathbb{E}[C^n]$ is minimized by buying D units of energy on the date with the lowest expected price, which is often the spot price on day n. Thus, we need a second constraint to avoid the risk that this choice involves. As in Section 4 we let the standard deviation of our income be a risk measure, and bound this risk by some α, i.e. $Var(C^n) < \alpha$.

Combining goals and constraints gives the following program

$$\begin{aligned} \min_{\xi_n} \mathbb{E}[C^n] &= \min_{\xi_n} \left(f_t^n F_t^n + \sum_{i=t+1}^{n-1} f_i^n \mathbb{E}[F_i^n] + s_n \mathbb{E}[S_n] \right) \\ \text{s.t.} \sum_{i=t}^{n-1} f_i^n + s_n &\geq D \ , \\ Var(C^n) = \xi_n \Sigma \xi_n^T &\leq \alpha \end{aligned} \tag{1}$$

where Σ is the covariance matrix for the random variables $F_t^n, \cdots, F_{n-1}^n, S^n$. Note that the variance of F_t^n is zero, as it is known. As a solution of this program we obtain the optimal strategy vector ξ_n of how many future contracts to buy on each day. The energy trader can now choose to buy f_t^n future contracts today and solve the problem again on day $t+1$ with adjusted values for the estimates.

The program (1) has several nice properties. Its objective function is linear, while the feasible region is a convex set. As the expected energy prices are non-negative the optimal solution will not exceed the demand D. Thus, enforcing an equality on the second constraint will not exclude an optimal solution, and we can reduce the feasible region to a compact set. Then we know there exists an optimal solution within this compact set as the objective function is linear. We can find this solution by using an algorithm for convex optimization (e.g. SPDT3 [5]). Such algortihms usually give good results and have polynomial complexity. Hence, larger versions of the problem can be solved without increasing the computation time too much, making the method fitted for large scale real life applications.

To actually run the given program, estimates for future and spot prices, and their respective covariances are needed. The quality of the solution depends heavily on the quality of these estimates. Note that the solution of (1) can involve negative values for the f_i^n's, and if this kind of price speculation is not wanted $\xi_n \geq 0$ can be added as an extra constraint.

To show how the convex optimization approach works we calculate an optimal strategy for an example case. Assume we are on the 30.1.2007 and we want to buy energy for 1.2.2007. This means that we can buy future contracts for that day either today or tomorrow, or we will have to buy energy at the spot price. Today's future contract price is $F_{30.1.2007}^{1.2.2007} = 29.25$, and further we find the estimates $F_{31.1.2007}^{1.2.2007} = 27.48$ and $S^{1.2.2007} = 27.33$. For simplicity assuming independence, we obtain the covariance matrix

$$\Sigma = \begin{pmatrix} 77.49 & 0 \\ 0 & 2.1 \end{pmatrix} ,$$

and as the demand we choose $D^{1.2.2007} = 1000$. Finally, we set the risk threshhold at $\alpha = 1000^2$. In this case, the described program will return the strategy

$$\xi_{1.2.2007} = (300.71, 679.32, 19.96) \ .$$

That is, we are told to buy a future contract for 301 units of energy today, a contract for 679 units tomorrow, while not hedging 20 units and buying these at the spot price. Accordingly, we buy a future contract for 301 units of energy today, and tomorrow we rerun our program with the updated demand $D^{1.2.2007} = 699$ and a new estimate of the spot price $S^{1.2.2007}$.

6 Conclusions and further directions

With a simplified model of the electricity market, we have successfully implemented models that find optimal short-term strategies for electricity retailer companies seeking to lower risk while keeping a decent profit. Backtesting shows that our models are making good predictions on a company's gain, and are suggesting decent strategies. However, our models can still be expanded in a number of ways, and we finish off by discussing some of these.

6.1 Modelling stochastic demand

An interesting model improvement to start with would be the modelling of stochastic demand. The demand is far from constant, as we have considered it, and is positively correlated with the spot price. During EMCI's Modelling Week, the demand data we were given were inaccessible to us due to the size of the data. However, with these data at hand one could easily forecast the demand in similar ways as we did the future contract and spot prices.

6.2 Options and long-term future contracts

As an assumption we have been working only with daily future contracts. A natural next step would be to work weekly, monthly or yearly future contracts, or options, into our model. With these derivatives at hand, an electricity retailer company would be able to reduce its risk significantly. In particular, under stochastic demand, entering into an option contract would be a safe way to deal with a forecasted peak in the demand.

6.3 Improving forecasts

All attempts of optimizing the trade strategy depend heavily on our ability to make accurate forecasts on the development of the derivative and spot prices, and of the demand. Hence, it would be favourable to improve our current estimates of these. A vast number of methods of forecasting are available and should be experimented with; time series analysis, neural networks or advanced econometric models to name a few.

Thanks

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References

[1] http://www.nordpoolspot.com

[2] Restructured electrical power systems , Shahidehpour, Mohammad 2001, chapter 9

[3] Options, futures and other derivatives, Hull, 6th edt, pages 1-11, 48-50, 554-555

[4] The mathematics of financial derivatives - a student introduction, Wilmott, 2002, page 14-15]

[5] http://www.math.nus.edu.sg/~mattohkc/sdpt3.html, SDPT3 – a MATLAB software for semidefinite-quadratic-linear programming