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Project: *Approximating cornea shape*

Report

List of persons participating in problem solution and report preparation

Anders Heyden, Kamil Bogus, Janusz Dagiel, Tomasz Kulpa, Marek Rogala, Filip Stachura

Abstract

The rapid development of technology allows people to repair the imperfections of the human body more precisely than ever. Almost every problem with eyesight caused by aberration in cornea shape could be repaired by contact lenses or even laser surgery. To do that we need method of determining what is an 'aberration' upon data obtained by keratoscopy. Vast amount of data obtained in every such test have to be then processed and examined. Therefore methods of compressing that information should be developed.

In our study we tried approximating corneal shape with an ellipsoid and compressing the remaining difference with image compression algorithms. We have developed method of compressing data to nearly 10% of original amount, while not going beyond given margin of error. Furthermore, we have improved methods of verification our algorithms. This report can be a prelude to more advanced study on image processing used in the subject of corneal shape modelling.

1 Introduction

Thanks to a very rapid development of technology nowadays, we are able to study the construction of many elements of the human body, such as the eye. Yet still we are not able to precisely model many of its elements, such as the surface of the cornea. Any success in this task would offer many practical applications in the field of contact lenses production, laser surgery or more general the examination of the nature and easier treatment of various eye diseases. This project is aimed precisely to find a suitable model of the human corneal surface. This report describes the results obtained by us and conclusions.

2 Problem analysis

Main subject of this study group was to prepare new methods to describe cornea surface. Data acquired from keratoskopy is a set of points (their polar coordinates and height). In this model, we have to store three values of type "double" for every point, around 9000 points for picture. While examining changes of cornea in time, there is around 150 pictures from one exam. This amount of data is hard to store and process, new model should be proposed.

First step is to correct irregularity in data. Points of measurment are located on 30 angles, and approximately on circles. After interpolation instead of set we have height values for defined grid (in polar coordinates).

Next step is to fit simple surface into this cornea and approximate difference between them. Simplest approximation is by fitting sphere (solving least square problem for 4 parameters).

$$\begin{bmatrix} x_1^2+y_1^2+z_1^2 & -2x_1 & -2y_1 & -2z_1 & 1 \\ & & \vdots & & \\ x_n^2+y_n^2+z_n^2 & -2x_n & -2y_n & -2z_n & 1 \end{bmatrix} \begin{bmatrix} 1 \\ x_0 \\ y_0 \\ z_0 \\ R^2+x_0^2+y_0^2+z_0^2 \end{bmatrix} = 0 \quad . \tag{1}$$

Where:

x_0, y_0, z_0, R are parameters of ellipsoid.

x_j, y_j, z_j cartesian coordinates of measured points.

Next one is fitting ellipsoid (solving least square problem for 10 parameters).

$$\begin{bmatrix} x_1^2 & y_1^2 & z_1^2 & x_1y_1 & x_1z_1 & y_1z_1 & x_1 & y_1 & z_1 & 1 \\ & & & & & \vdots & & & & \\ x_n^2 & y_n^2 & z_n^2 & x_ny_n & x_nz_n & y_nz_n & x_n & y_n & z_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \\ j \end{bmatrix} = 0 \quad . \tag{2}$$

Where:

$a \dots j$ are parameters of ellipsoid.

x_j, y_j, z_j cartesian coordinates of measured points.

Last step is to use known image compression algorithms. For regular data - values in every point of a grid - it is possible to describe it as an 2-dimensional image. Difference between compressed image and original is measured with Root Median Square

$$x_{RMS} = \sqrt{\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}} \quad . \tag{3}$$

Where $x_1, x_2, \dots$ are differences between original and compressed image. Precision that we have to achive is RMS of error smaller than 1 micron.

One of most important things in examining karatoskopy results is curvature of the surface. Omitting the azimuthal component, the curvature of the cornea at a given point (r, θ) can be given as:

$$k = \frac{\frac{\partial^2 z}{\partial r^2}}{[1 + (\frac{\partial z}{\partial r})^2]^{\frac{3}{2}}} \quad . \tag{4}$$

Surely, it have to be preserved during compression.

3 Data interpolation

Using spline interpolation in polar grid points we acquire set of normalized data.

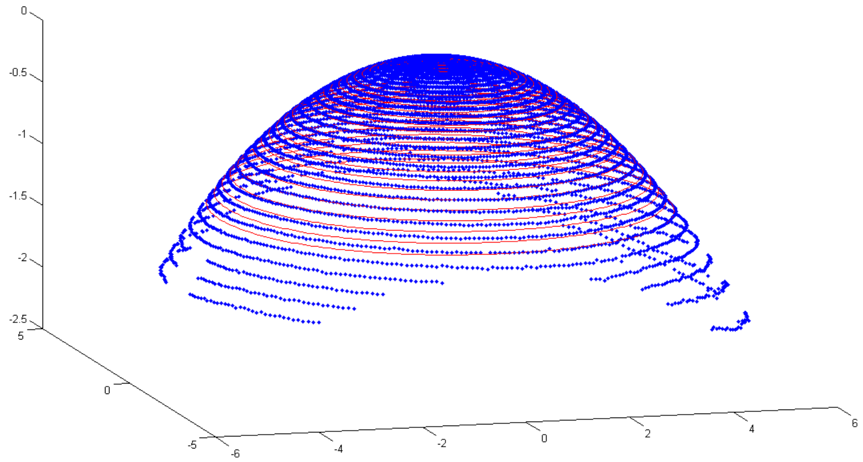

Figure 1: Blue dots - samples; since now we are using interpolated values on red rings

It is stored as three numbers determining: distance between rings, number of rings (usually 35) and number of angles (usualy 300 angles) and a vector of heights. After normalization data is already compressed three times (from $3 \times n$ to $n + 3$).

4 Compression results

After converting height values (in range of 2-3 milimeters) to one of 256 colors, result could be compressed using image compression methods. Our group tested four different methods, with following results:

»compress_image(I_h,'png');		»compress_image(I_h,'jpeg');	
Size: 300 x 30		Size: 300 x 30	
Min:	-0.002370	Min:	-0.016641
Max:	0.002370	Max:	0.015389
Mean:	-0.000066	Mean:	-0.000198
RMS:	0.001293	RMS:	0.004779
Compression rate:	34.026465	Compression rate:	29.801325
»compress_image(I_h,'gif');		»compress_image(I_h,'bmp');	
Size: 300 x 30		Size: 300 x 30	
Min:	+0.000000	Min:	-0.002370
Max:	1.208636	Max:	0.002370
Mean:	0.807657	Mean:	-0.000066
RMS:	0.883607	RMS:	0.001293
Compression rate:	37.656904	Compression rate:	3.371418

Table 1: Compression alghoritms comparison

As seen, best compression ratio with low RMS of error is for PNG compression. Although comparison of curvature maps between compressed and originals shows, that this method shoud be modified.

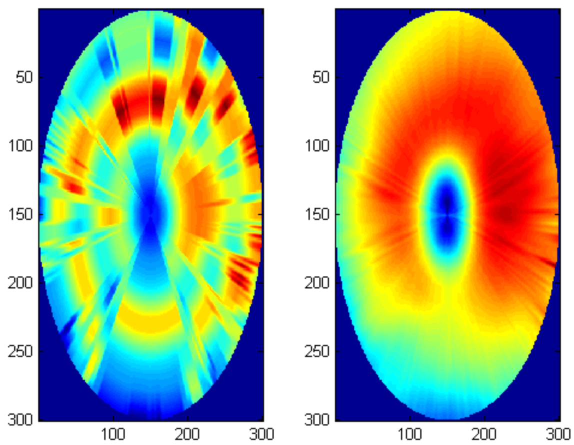

Figure 2: Comparison between compressed image (left) and original (right)

It was done by narrowing range of heights by substracting ellipsoid from cornea. Differences were in range of 50 microns, then transforming them into 256 colors do not cause such information loss. This method also provided satisfying compression results. Below are statistics for four different types of cases provided.

»compress_image (NormalEye_MinusEllipsoid,'png');	»compress_image (Astigmatism_MinusEllipsoid,'png');
Size: 300 x 30	Size: 300 x 30
Min: -0.000029	Min: -0.000198
Max: 0.000029	Max: 0.000198
Mean: 0.000000	Mean: 0.000003
RMS: 0.000017	RMS: 0.000115
Compression rate: 9.779951	Compression rate: 16.861827
»compress_image (ComplexCase_MinusEllipsoid,'png');	»compress_image (Keratoconus_MinusEllipsoid,'png');
Size: 300 x 30	Size: 300 x 30
Min: -0.000158	Min: -0.000094
Max: 0.000158	Max: 0.000094
Mean: 0.000001	Mean: 0.000000
RMS: 0.000090	RMS: 0.000054
Compression rate: 13.095671	Compression rate: 11.714937

Table 2: PNG compression for different types of cornea.

This methods were checked on all test data provided. Below are statistics for PNG compression with and without ellipsoid substraction.

	without ellipsoid substraction	with ellipsoid substraction
Mean RMS (microns)	0.4635053	0.01446613
Median RMS (microns)	0.4547789	0.008944224
Mean compression rate	35.86252	10.55365
Median compression rate	32.62266	10.42270

Table 3: Overall compression results

As seen above, substracting ellipsoid helps to preserve high precision. Although, compression rate is three times lower.

In the following figure, we can see comparisons of curvature maps between original and compressed datasets.

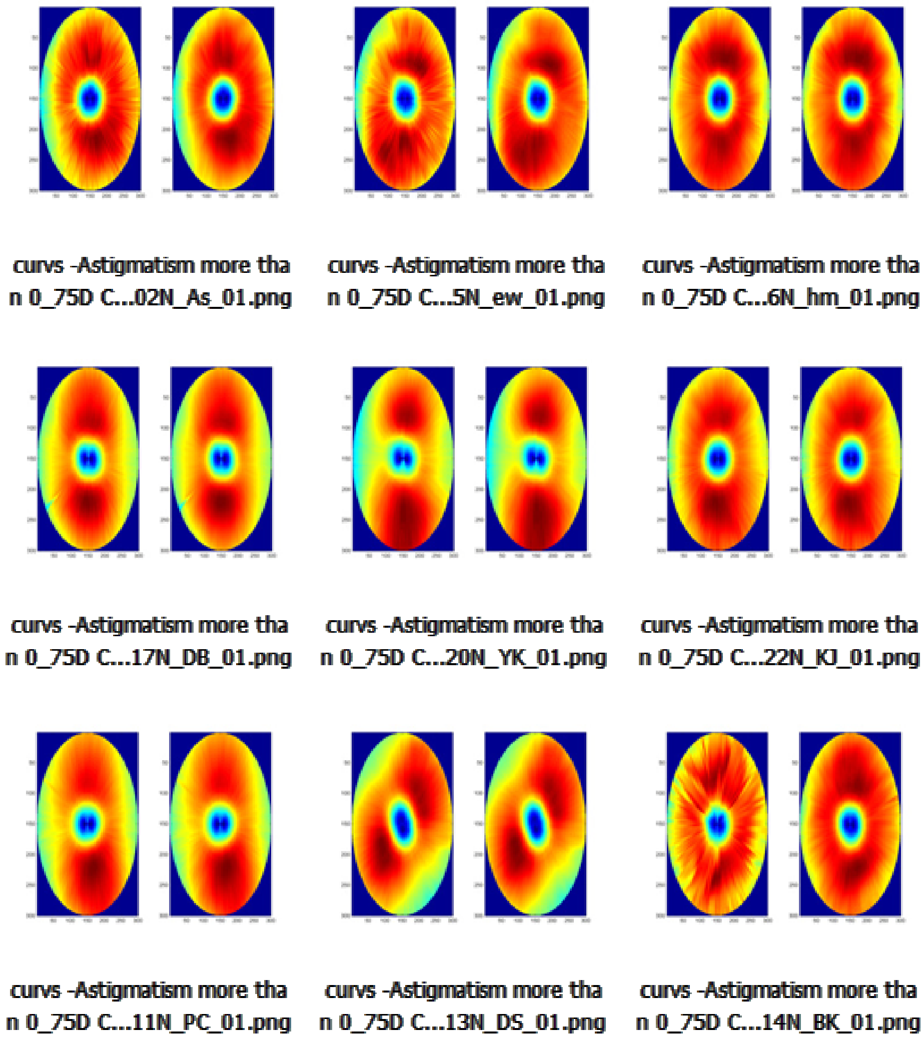

Figure 3: Comparison of curvature maps for compression after substracting ellipsoid. Original image (on the right in every pair) and compressed (left)

This method preserve curvature which is one of the most important parameters in analysing cornea distortions. Therefore, in contrast to simple compression, it could be used in this case. Although, as visible in last presented case, this method is imperfect. While RMS much smaller than 1 micron, which is what we had to achieve, some information is lost during compression. This enforces to include differences in curvature in our determinants of algorithm corectness.

Our group performed short experiment with fitting polynomial to difference between ellipsoid and cornea surface. Polynomial of 7th order had not increased significantly precision or compression rate. This way of research was abandoned, small aberrations are too subtle to be included in polynomial. Although finding class of functions that would do better is one ossible way of further investigation.

5 Conclusion

As shown, it is possible to compress data obtained during keratoskopy without losing precision. Normalizing data, fitting an ellipsoid into cornea surface and compressing difference using PNG image compression allows us to compress data 30 times. This compression rate is achieved at cost of 0.3 micron loose in precision (maximum difference between original and compressed image). It is worth to mention that proposed alghoritm preserves curvature in most cases - one of important parameters in examining cornea aberrations.

One path of further research would be approximating difference between ellipsoid and cornea surface with functions more sophisticated than polynomials. Another would be to analyze and modify PNG alghoritms used to compress images that they would suit our needs.

A Scripts

There will be included scripts used if we decide to include them.