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Project: *MILK*

Report

Helmut Neuzert, Tomasz Jetka, Agnieszka Jurlewicz,
Agnieszka Kaszkowiak, Jakub Nowotarski, Paweł Szerling, Paweł Wilk

1. Content

1 Introduction

1.1 Problem given by the industry

The farmer — Wojciech Wawrzyniak — the owner of milking cows wanted to get the best quality milk and save money on electric energy. He remarked that due to the long duration of the milk cooling process, the quality of the milk is decreasing. It results in a decrease of the price of the milk and therefore in a drop in the farmer's income. Of course, the farmer wants to maximize the income from milk sold.

On the other hand, the farmer wanted to save money on energy. Electricity is used mainly by cooling device and water heater. Thanks to the heat exchanger some amount of heat can be retrieved.

1.2 Case study description

The farmer needs a solution, how to optimize the cooling process. The most important processes which project team has taken into account were: milking, cooling and bacteria growth.

1.2.1 General information about the farm.

There are 70 milking cows on the farm. Each cow gives approximately 11,5 liters of milk in 1 milking. In the same time 10 cows could stay on the milking platform. The minimal time interval between milkings equals 15 minutes. The milking of a cow lasts less than 2 minutes, but it takes about 7 minutes for a cow to enter and descend from the milking platform.

According to the data obtained from the farmer, energy costs are only 4% of total income and 6,25% of total expenditures. Because of such a small importance of them, the project group has not concentrated on energy cost optimization in further works.

1.2.2 Influence of milk quality on the prices

The number of bacteria has an influence on the quality of the milk. The more bacteria in milk, the lower price of 1 liter of milk. The table below presents appropriate prices of milk depending on its quality.

Number of bacteria in 1 ml of milk	Price of 1 l of milk
<= 50 000	1,19 PLN
> 50 000 and < 100 000	1,07 PLN
>= 100 000	0,30 PLN

Table 1. Prices of 1 liter of milk

Source: Farmer's data.

Quality of milk depends on seasonality. Average number of bacteria in 1 ml of milk during cold months equals 20 000. During warm months the average is even higher than 30 000. Obviously, in summer there is higher probability of high level of bacteria because of longer time of cooling.

Rapid and efficient cooling is essential for preserving milk quality. Fresh milk leaves the udder at approximately 35°C and the heat must be quickly removed. Milk retains a natural resistance to bacteria immediately after extraction, but only rapid cooling to a storage temperature of around 4°C to 6°C, prevents further micro-organism growth.

1.2.3 Milking and cooling process

The milking as a part of dairy management is a procedure, where the milk is drained from cows. It is also an occasion when the farmer has the opportunity to control and observe the cow. During one milking period, 10 cows are connected to milking machines and produce milk with temperature of ca. 33-35°C.

After the milking, the milk taken from 10 cows is delivered to the cooling machine. While the milk is being cooled, next 10 cows are milked. Afterwards, a new milk portion is added to the cooling machine and the process iterates until all cows are milked and whole milk is cooled. There are 2 milkings each day.

The modern way of milk cooling uses electricity to generate the temperature required. '*The electricity runs the condensing unit, which condenses the evaporated liquid and makes the process a continuous cycle. Cooling systems transfer the heat of the milk via a cooling agent to either air or water. This transfer goes via a separated wall, so there is never direct contact with the milk. The refrigerant, or cooling agent absorbs the heat of the milk inside the evaporator. The final temperature depends on the thermostat setting or milk flow through the plate coolers. Large differences in temperature increase the rate of cooling. High speed and turbulent motion of liquids along the wall will improve the heat transfer rate*'[1].

1.3 Identification of the problem

Due to extremely low share of energy in the total costs and the inability to modify the power consumption during the processes, the project group has rejected a reduction in energy costs as a way to solve the problem.

The objective of improving the quality of milk by reducing the number of bacteria was considered a key issue during the design work. The team found that the possibility of controlling the process lies in modifying the time of milking and cooling.

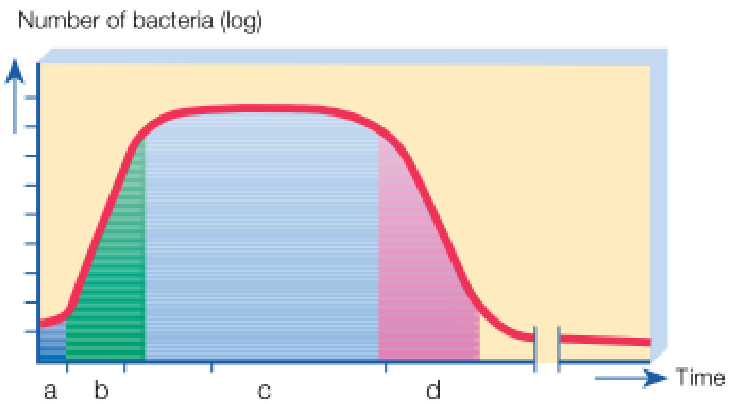
[1] http://www.delaval-us.com/Dairy_Knowledge/EfficientCooling/Cooling_Technology.htm?wbc_purpose=htt (link active on 2010/10/31)

2 Theoretical background

In order to find an optimal strategy for the organization of milking we must thoroughly examine the core processes which occurs in this situation. Our main interest is to reduce the ultimate volume of bacteria in milk. Hence, we will firstly look into the models which describe the growth of the milk bacteria. Since the temperature is identified as the most important factor in such cases, we will investigate into the theoretical background which lies beneath the cooling process. The information provided in this section will be used to create a simulation which will evaluate the volume of bacteria in the milking process.

2.1 Bacteria growth process

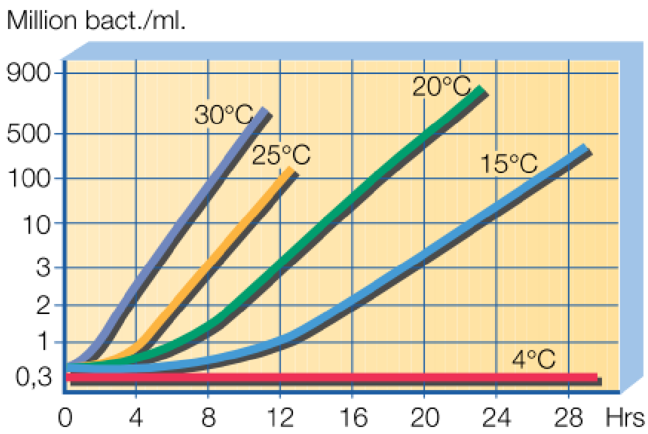
Well known biological theories state that bacteria reproduce by cell division. They grow to a fixed size and then reproduce through binary fission. Under optimal conditions, bacteria can grow and divide extremely fast. However, there are some restrictions which prevent them from unlimited reproduction. Hence, there are identified 4 phases of growth of colony of bacteria: lag phase, exponential phase, stationary phase, and death phase [1]. It is shown on the Graph 1:



Graph 1. The growth of bacteria colony. Phases of growth are shown:
a - lag phase, b - exponential phase, c – stationary phase, d – death phase.
Source: *Dairy Processing Handbook,* Tetra Pak, 1995.

Nevertheless, in our case we will focus on the initial stages (first 48 hours) of the bacteria growth only. Therefore, we can assume that we will face the exponential phase of bacteria growth and turn to the Arrhenius or Vogel-Fulcher-Tammann empirical equations to model the rate coefficient of this process [2].

As far as milk bacteria are concerned, one of the essential factors to their growth is the temperature. The higher the temperature, the faster bacteria (in which we are interested) will reproduce. In addition, it is empirically proven that they tend to stop to growth at temperature of about 4°C. Graph 2 shows the likely dependence of the temperature on the bacteria growth.



Graph 2. The influence of temperature on bacteria growth.
Source: *Dairy Processing Handbook,* Tetra Pak, 1995.

Taking into account all of that, the following equations will be used to simulate the bacteria changes in time:

$$\frac{dB}{dt} = \alpha(T)B(t)$$

$$B(t) = B_0 e^{\int_0^t \alpha(T(s))ds}$$

$$\alpha(T) = ce^{-\frac{A_0}{T - T_\infty}}$$

Where:

t - the point in time
$T(t)$ - temperature in time t
$B(T)$ - volume of bacteria in time t
B_0 - the initial volume of bacteria
$\alpha(t)$ - rate of growth of bacteria
c, A_0 - growth coefficients
T_∞ - limit temperature

Coefficients which appear here have been estimated using the Least Square Method:

$c = 2,53$

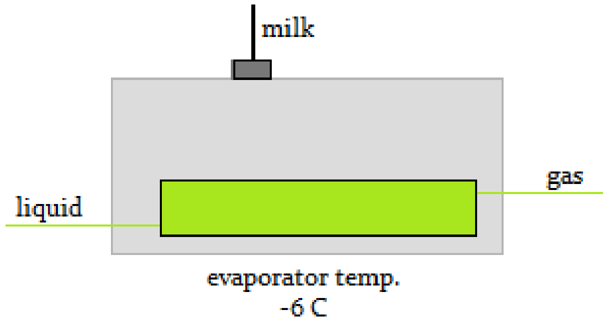
$A_0 = 33,72$

The limit temperature is set to $T_\infty = 4$.

2.2 Cooling process

As it was stated in previous section, the temperature is essential factor for the bacteria growth. Because of that, it is important to look into details of the milk cooling process.

Let us briefly remind the main facts about it: the initial temperature of milk is about 35°C, it should be cooled to 4°C which is the optimal temperature to preserve milk (both lower and higher temperatures could negatively affect the quality of milk). The scheme of the cooler used is presented below:



Graph 3. The system of cooling.

Source: Farmer's information

The cooling device is situated at the bottom of the milk container. In the process of cooling the method used in refrigerators is applied. Precisely speaking, the heat is taken by the evaporation process. Specific liquid goes through the cooler, takes heat from the milk to evaporate, and as a gas goes out from the system. One of important information is that because of the cooler position, the cooling area is always the same. Hence, the heat that can be taken by the cooling device at given time does not depend on the volume of milk [4,5].

According to this information we can assume that the process of cooling is described by the following equation:

$$T(t) = T_0 - \frac{\gamma_s}{m} \cdot Q_{evp} \cdot t$$

Where:

$T(t)$ - temperature in time t

T_0 - initial temperature

γ_s - evaporation coefficient

m - mass of the milk

Q_{evp} - heat taken through evaporation

There are two main features connected with the equation above:

1. The time of the cooling (to given temperature) is inversely proportional to the mass of the substance cooled.
2. Suppose that there is a specific volume of milk in the container, already cooled into the desired optimal temperature. We add next portion of milk, which has the temperature of 35°C. Then the time of the cooling is independent of the volume of milk which has been inside before (there should be provided specific amount of heat which depends only on the mass and temperature of added milk).

3 Model

In order to forecast the probable changes in quality of milk resulting from the reorganization of milking and cooling process, the project group has created a mathematical model of this problem. It is based on theories presented in Section 2. Consequently, number of simulations have been carried out to obtain possible solutions.

3.1 Assumptions

Following assumptions have been made about the cooling, milking and bacteria growth process:

Milking process

1. Whole process of changing, milking and cleaning of group of ten cows takes at least 15 min.
2. Milking process of group of 10-cow is immediate.

Cooling process

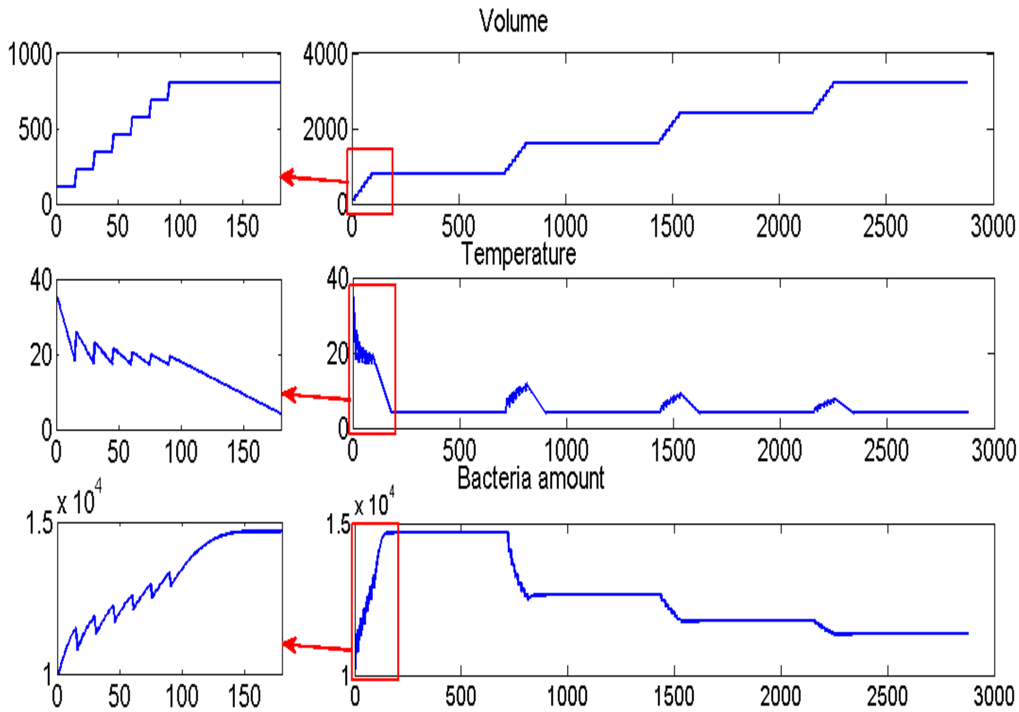
1. In the storage the heat flow between milk and cooling device is constant whenever the temperature of milk is above 4°C. When temperature of milk reaches the temperature of 4°C, the heat flow disappears.
2. In the storage the mixing milk process is immediate. Hence, soon after the new portion of milk is added, all milk in the storage reaches the same temperature.
3. The cooling process of whole storage of milk (3200 l) from 35 to 4°C takes 720 min.

Bacteria growth process

1. At milking instant of time the number of bacteria is equal to 10 000[ml^{-1}].

3.2 Simulation

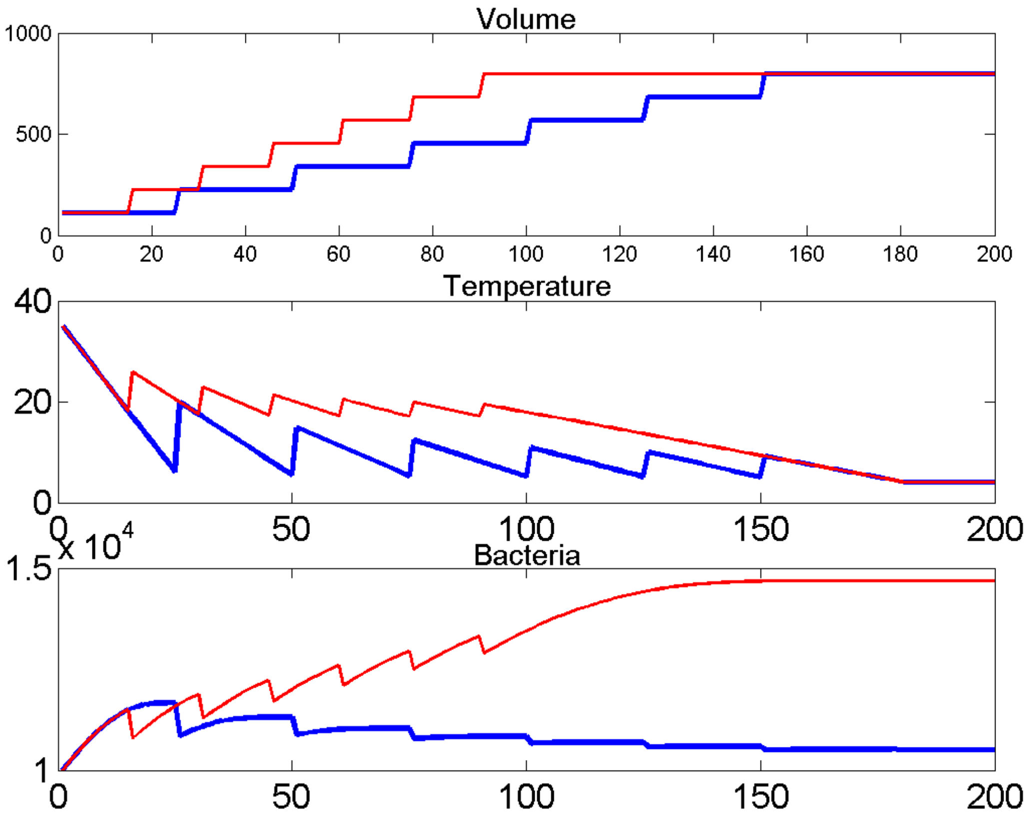
Firstly, let us focus on a general 2-day perspective, as it is the case of the milkman. As mentioned in Section 1, there are four milkings (two per day), and each of them consists of 800l of milk. Limited by technology, we divide the amount of milk from one milking into seven groups. Basing on the theory presented in section 2, we have made a simulation in Matlab and as a result we have obtained the following graphs:



Graph 4. The 2-day process perspective, together with the details on the first milking process.

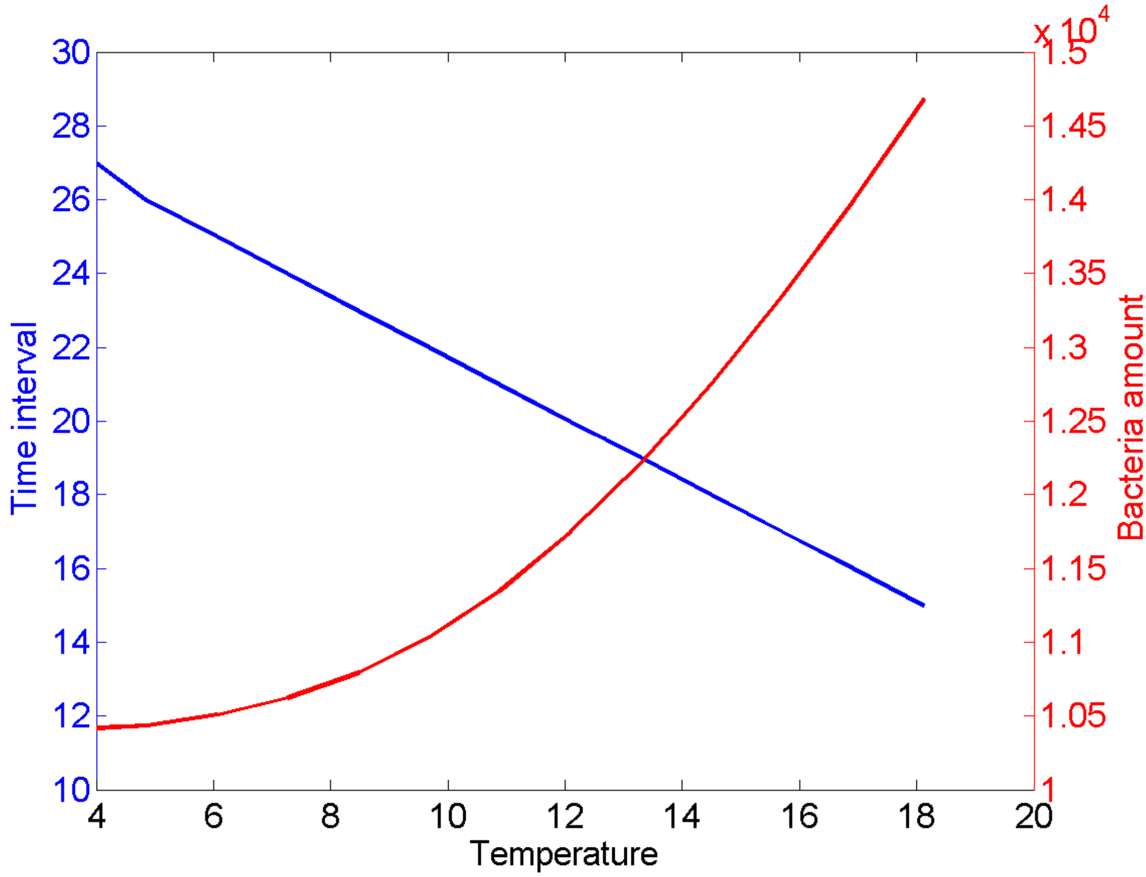
They show three major values in milking-cooling process: volume of milk in liters, temperature in Celsius degrees, and number of bacteria per milliliter. The x axis represents time in minutes in all six plots. On the left we see three plots showing the beginning of the process in zoom with the time on the x axes reduced to around 180 minutes (more or less the length of the first milking).

The analysis of graph 4 shows that the first milking-cooling process is of a crucial importance for the final quality of milk. Therefore, going on with studying the milking strategy we focus only on the first milking. Our idea is to find a relationship between the final number of bacteria in the milk and a time interval needed for each 10-cow part of milking. Hence, our next step is to compare and study two milking strategies, see Graph 5. In the first (red) the 10-cow milking time is 15min, in the second (blue) – 25min.



Graph 5. The focus on the first milking: comparison of 15min (red) and 25min (blue) strategies for 10-cow milking.

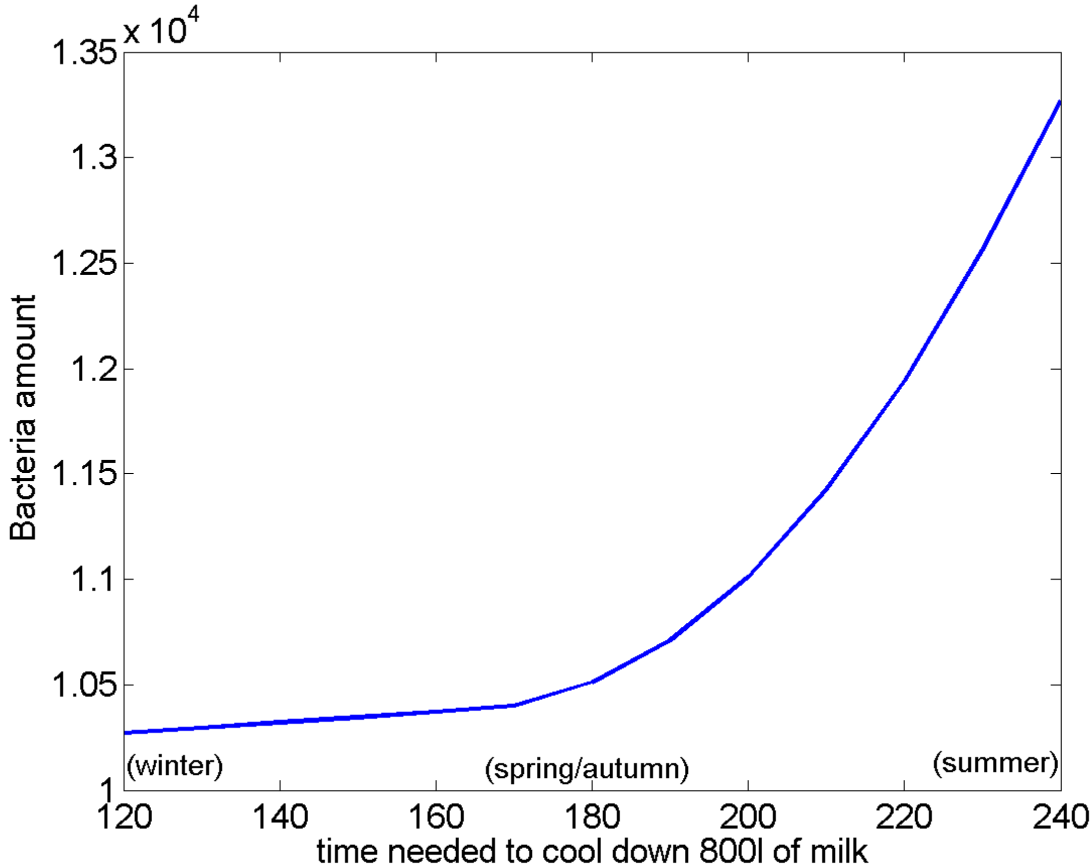
In graph 5, x axes show time in minutes, limited to 200 minutes because above that value we are sure that the first milking process is finished. In the picture there are three plots. As previously, at the top there is the volume, then the temperature, and the number of bacteria at the bottom. It is easily noticeable that in the 25min case (blue) the milking process takes much longer, but the milk is almost all the time colder and, what is highly notable, the number of bacteria is much lower that in the 15min strategy (red). Inspired by this result, we make a step forward and plot another picture:



Graph 6. The 10-cow milking time interval and the bacteria amount dependence on the temperature to which the stored milk is cooled down to add next part of the milk (decision variable).

The x axis shows the temperature of the milk inside the cooler in Celsius degrees we want to reach in order to begin adding the next 10-cow part of milk to the cooler (our possible decision variable). There are also two y axis: the left one is for the 10-cow milking time, and the right one shows the amount of bacteria. The plot is made not only to find the optimal strategy for bacteria growth, but also to help the milkman find a strategy for himself, as he may not be interested in increasing the time of the milking process up to almost 3 hours. From the picture we can conclude that the longer we cool down the milk, the less bacteria we have at the end of the cooling process. However, this results in lengthening the process.

The last aspect that we cannot omit is the milking strategy dependence on a season. From very general data that we have received from the milkman, we know that the amount of time needed for cooling down all 800 liters (from one milking) can vary from 2 to 4 hours in different parts of year. Therefore, considering again one milking process divided into seven portions and the 25min 10-cow milking time interval, we have prepared Graph 6 that shows a function connecting the total time of milking (x axis) with the number of bacteria at the end of cooling process (y axis):



Graph 6. The bacteria amount dependence on the seasonality.

As we can observe, the problem of high level of bacteria is likely to occur during the summer.

4 Recommendations

Our main recommendation for the farmer is: "If you want to decrease the amount of bacteria in milk, you should make longer breaks between milking of successive groups of ten cows". It can be the most important strategy especially:

- In the summer when the time outside temperature is the highest and the cooling machine works less effectively that makes the cooling process longer, and hence causes a higher temperature of milk and a faster bacteria growth.
- During the first among four milkings as afterwards the cooled milk in the storage is used as a precooler for new milk.
- Between the first and the second groups of cows because during the milkings of next group of cows, a cooled milk in storage is used as a precooler for a new portion of milk.

Bibliography

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