MODELS FOR INDUSTRIAL PROBLEMS:

How to find and how to solve them – in industry and in education

Helmut Neunzert

Fraunhofer-Institute for
Industrial Mathematics ITWM

Kaiserslautern, Germany

Man pointing, MoMA, New York, Photo: S. Mährlein

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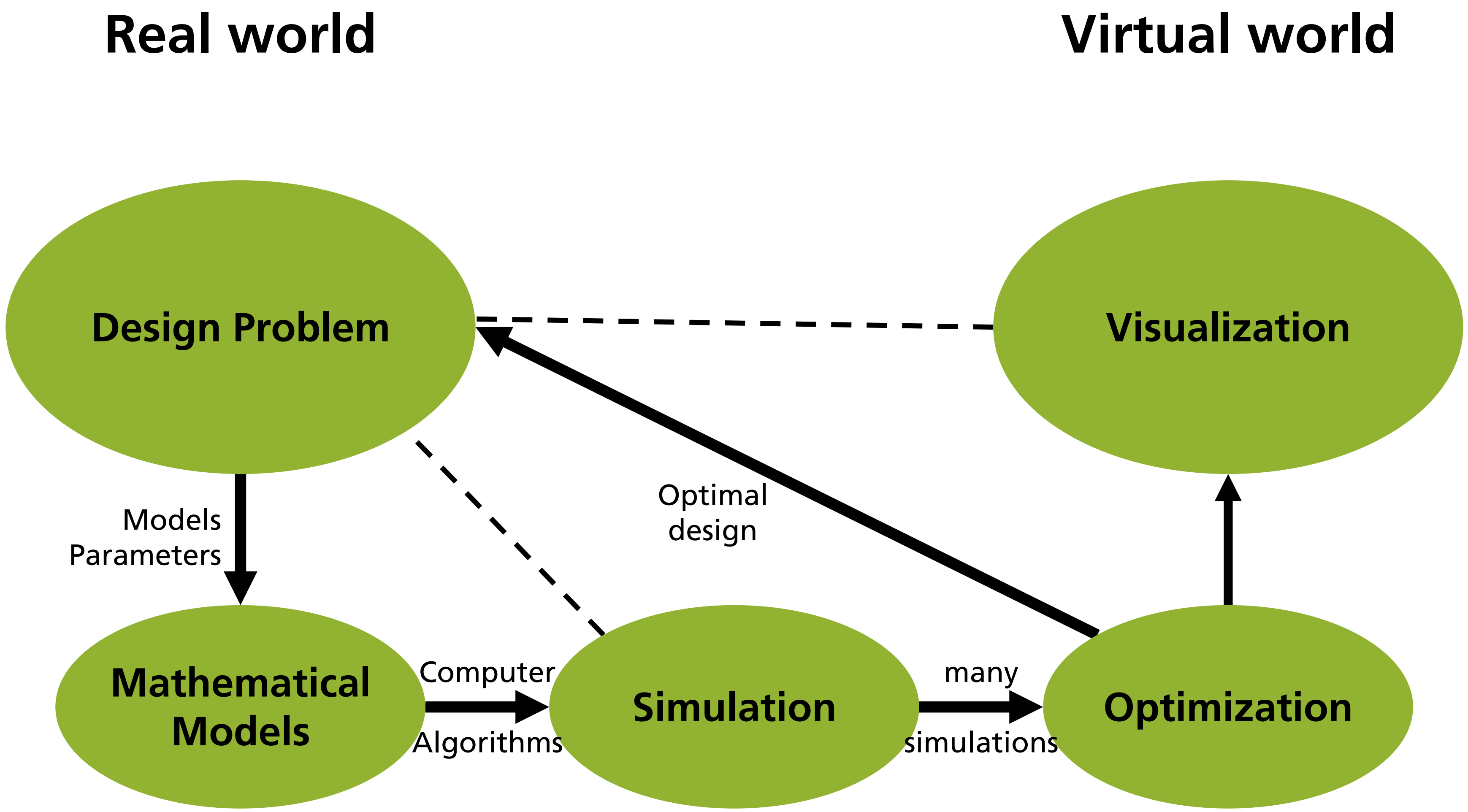
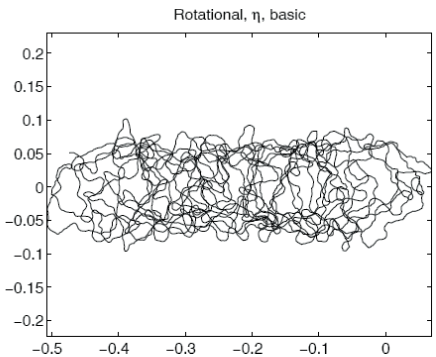
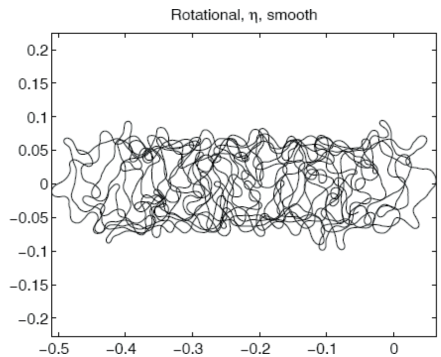
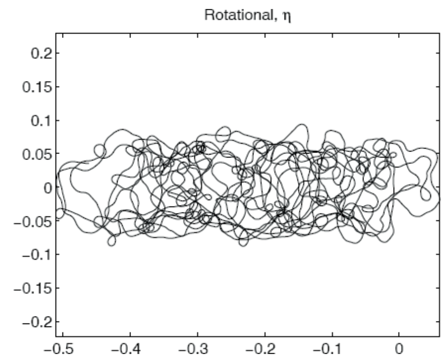

Real world
Virtual world
Design Problem
Visualization
Models
Parameters
Optimal
design
Mathematical
Models
Computer
Algorithms
Simulation
many
simulations
Optimization

Two Messages

- Keep in mind, that the model must be evaluated in a given time with given tools; this may be lead to a hierarchy of models







- Keep in mind, that the problem poser may change his mind, especially with respect to objective functions

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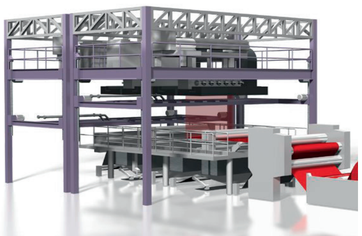
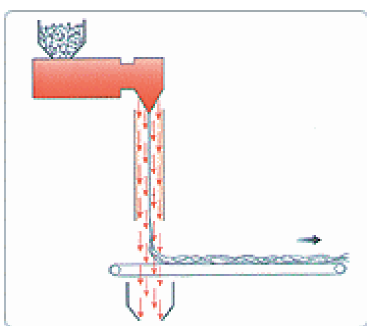
Example 1: A Hierarchy of mathematical models for production processes of technical textiles
(see A. Klar, N. Marheineke, R. Wegener; ZAMM 2009)





Granules

↓

Melting and spinning of fibers

↓

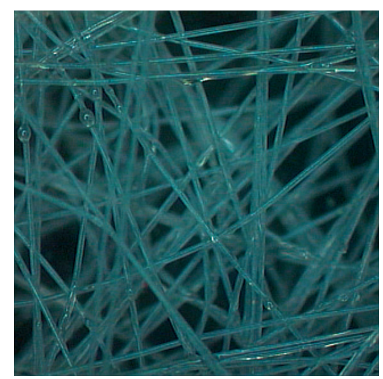
Curling of fibers through turbulences

↓

Deposition at the conveyor belt

↓

Non-wovens



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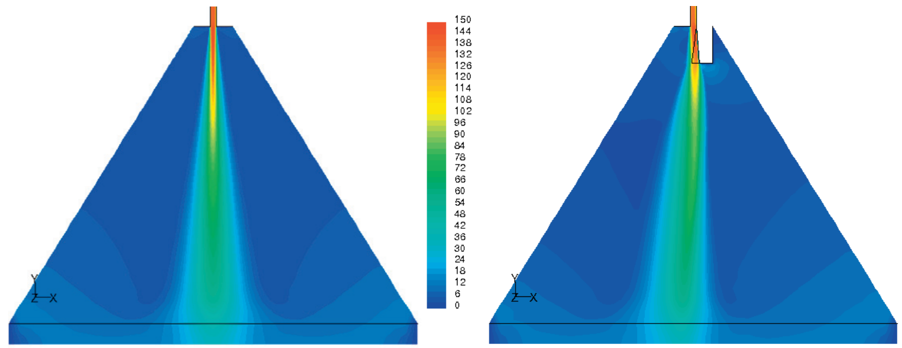
The input – output system

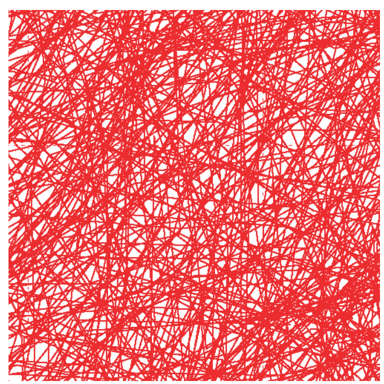
Input:

- fluid data
- geometry
- (material)

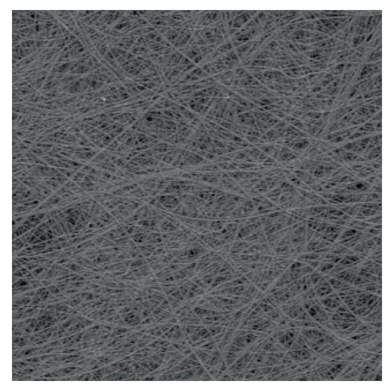


Output:

- quality of the fleece



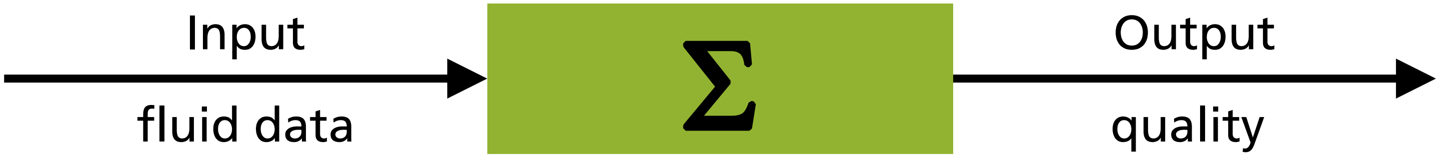
virtual fleece



real fleece

How to measure quality?

System theoretical description



Σ = class of mappings

Little theory ➔ class contains many parameters ➔ many observations are necessary in order to identify these parameters

Class: linear control systems, neuronal networks etc.

Black box models

Disadvantage: Only the prediction of already existing systems is possible

Much theory ➔ class contains few, i.g. measurable parameters

Class: Equations of continuum mechanics

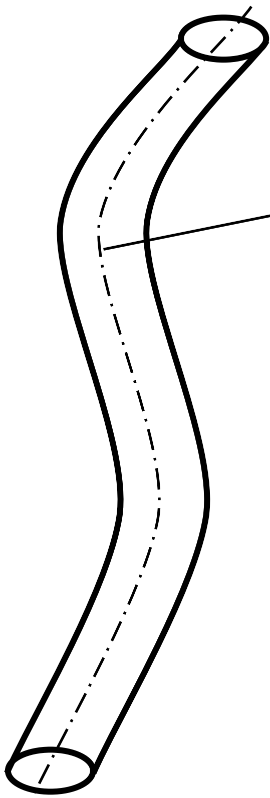
White box models

Disadvantage: Very costly numerical evaluation

In-between: **Grey box models**

The »almost white« model

Theory of Elasticity:



Rod theory (1d)

$\underline{r}(s,t)$

arc length ↗ ↖ time

$$\partial_{tt}\underline{r} = \partial_s(T\partial_s\underline{r}) + D_s\underline{r} + f$$

$\|\partial_s\underline{r}\| = 1$ Incompressibility

$f =$ Air forces

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Theory of Fluids

Navier-Stokes Equations which contain air forces f

$f =$ depend on the relative velocity

$\partial_t \underline{r} - u$

The numerical solution of Navier-Stokes in 3d with higher Reynolds numbers (10^4) is not feasible!

➔ This „almost white model" is not applicable.

Turbulence models (as k-ε model) need 2-3 hours per fiber: Still not applicable, since we have 1000 fibers.

➔ Further simplifications are needed, i.e. „British modeling" = The search for small parameters + asymptotic analysis.

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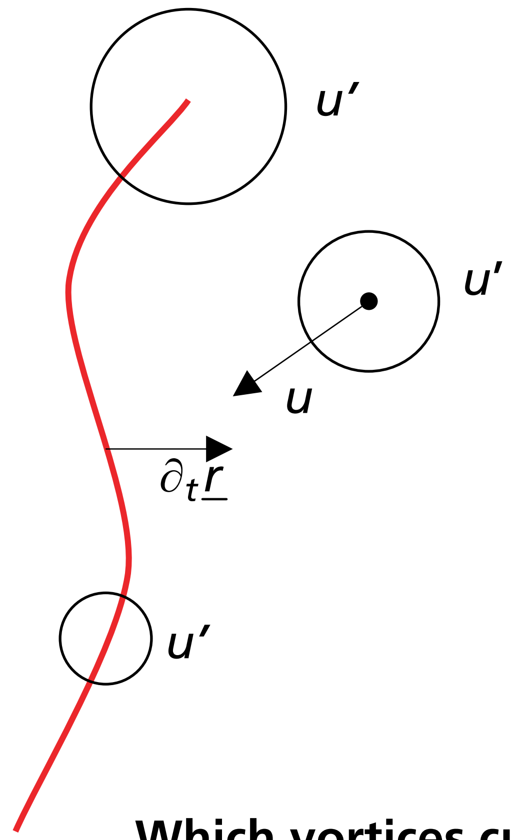
The length scales of the flow

fiber length $L \cong$ typical length 1

fiber thickness $\sim \sqrt{A}$



Turbulence contains vortices of different sizes, which interact with the fiber.

The vortices have different energies, depending on their diameters.

Which vortices curl the fibers?

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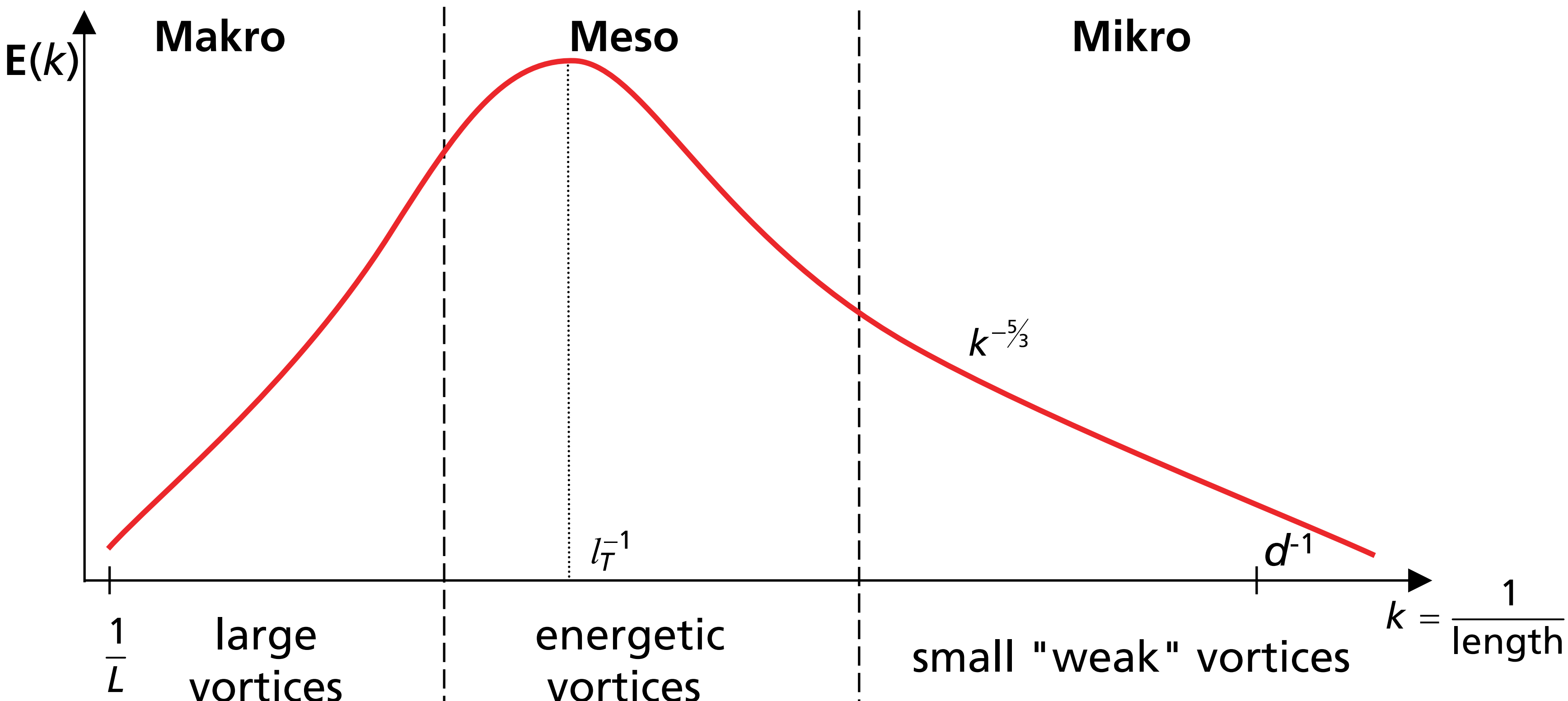
Kolmogorov-theory



Motion of fibers is determined by the bulk velocity

Interior forces and turbulent forces in equilibrium

No interaction

The asymptotic limit

The fundamental scale is l_T, which we compare with the length L of the fibre

$$\delta = \frac{l_T}{L} \approx 10^{-3}$$

Asymptotic limit: $\delta \to 0$

Then turbulence forces $\xrightarrow{\delta \to 0}$ white noise

Now, a simulation can be made, but takes still several hours for realistic situations

$\to$ No optimal process design is possible

Still too costly (2 – 3 hours)

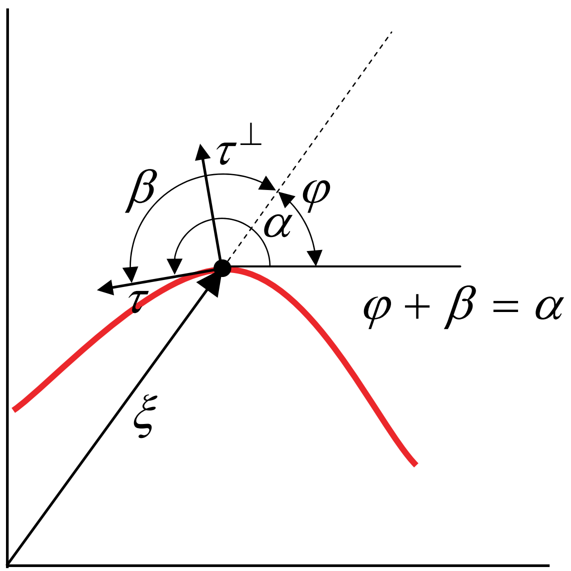
We have hundreds of fibers!

Further simplification

is needed = The **»grey model«** for the deposition

Non-moving conveyor belt „Impact point" of the fiber at the belt = $x\,(s)$

fiber incompressible è $t = s$



$$\dot{\xi} = \tau \quad , \quad \dot{\alpha} = -b(\|\xi\|) \underbrace{\frac{\xi}{\|\xi\|} \cdot \tau^\perp}_{\cos(90-\beta)=\sin\beta} + \tilde{A}dW$$

$0 \le \beta \le \pi \quad \Rightarrow \quad \sin\beta \ge 0 \quad \Rightarrow \quad \alpha$ decreases

$\Rightarrow$ The motion is turned towards direction ξ depending on b

$\tilde{A}$ = Amplitude of the projection of the turbulence

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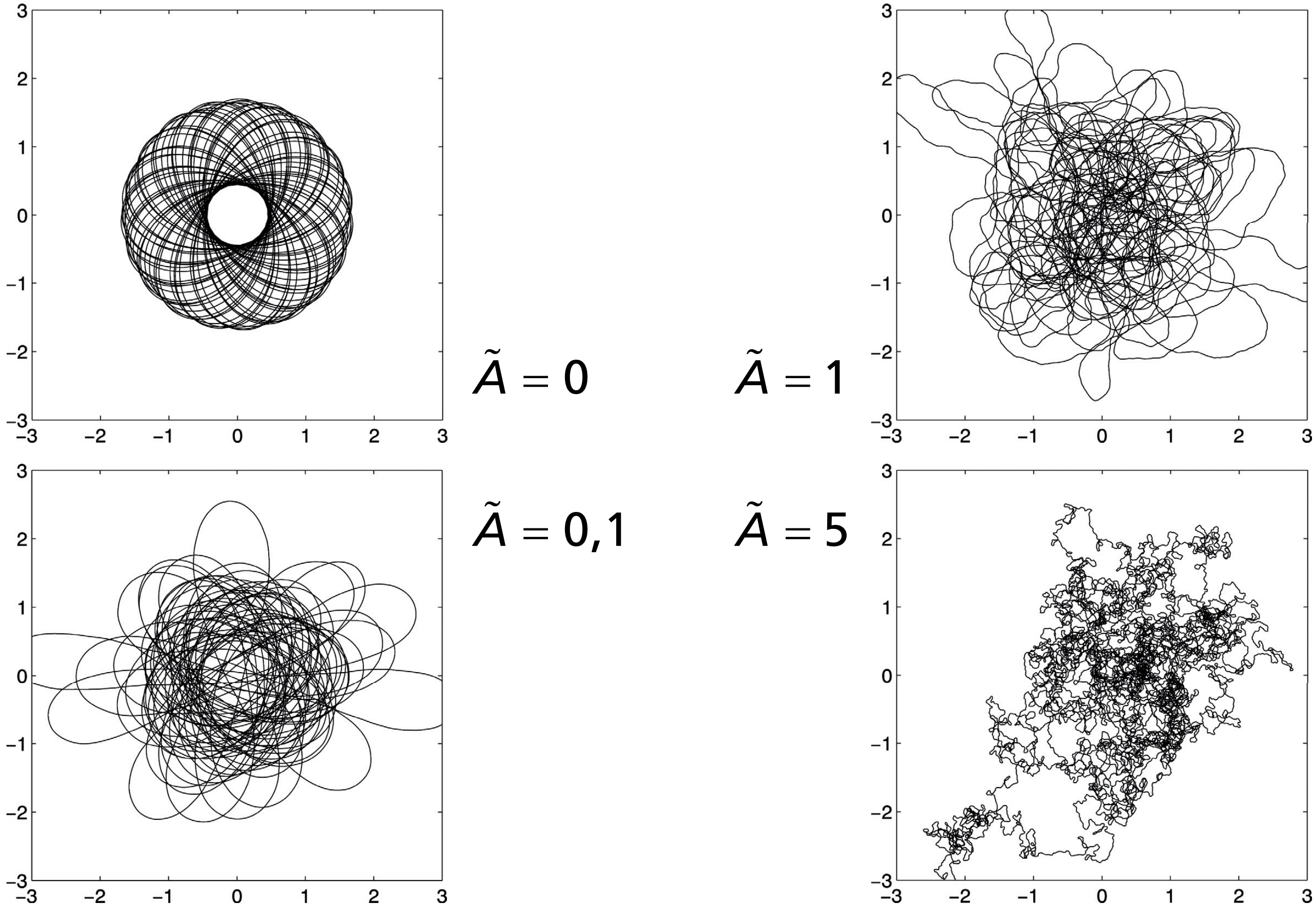
Influence of the turbulence

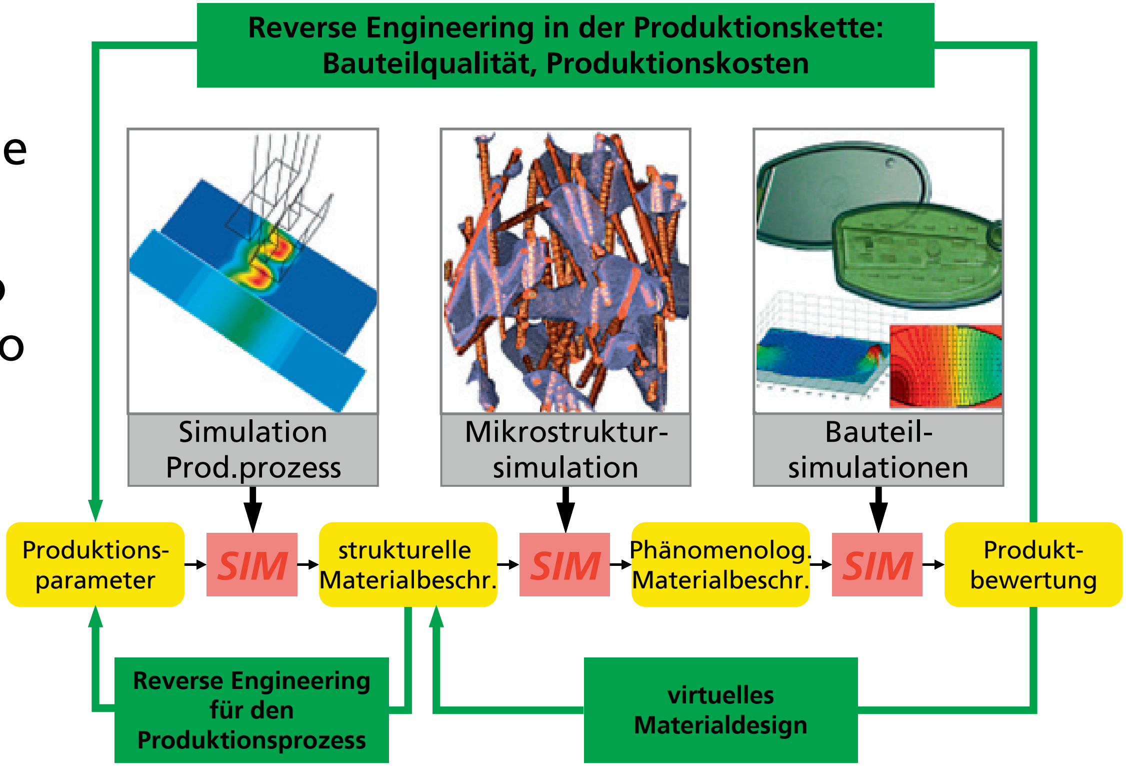
conveyor belt doesn´t move, $b(\|\xi\|) = 1$



$\tilde{A}$ and b are identifiable parameters.

A never-ending problem:

- Model and simulate the complete chain from production process to the final product
- Inverse problem: How to control the production to get an optimal product?
- Example: from filter features back to the spinning process



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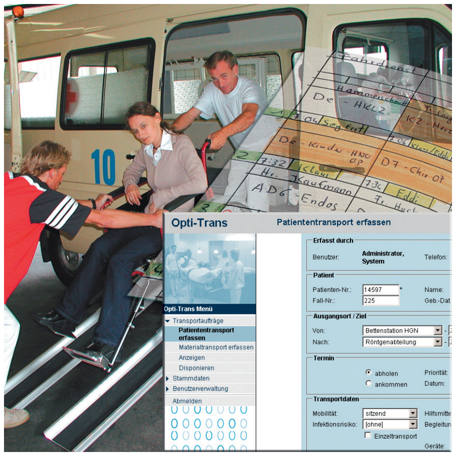
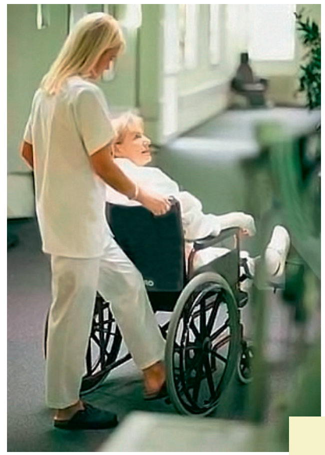
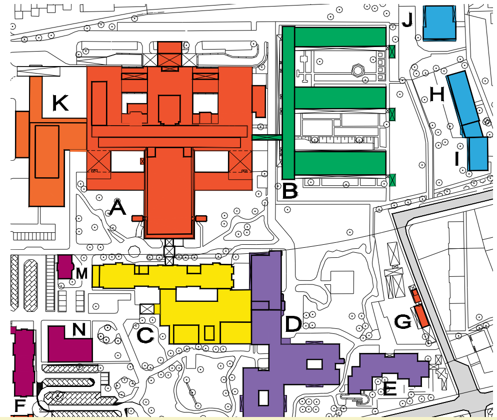
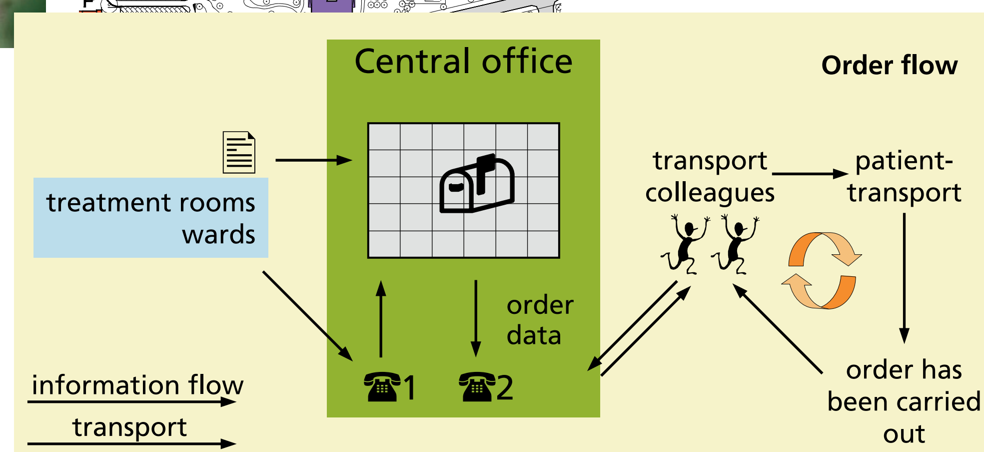
Example 2: Transport of Patients in Hospitals

(M. Schröder, Fraunhofer ITWM)









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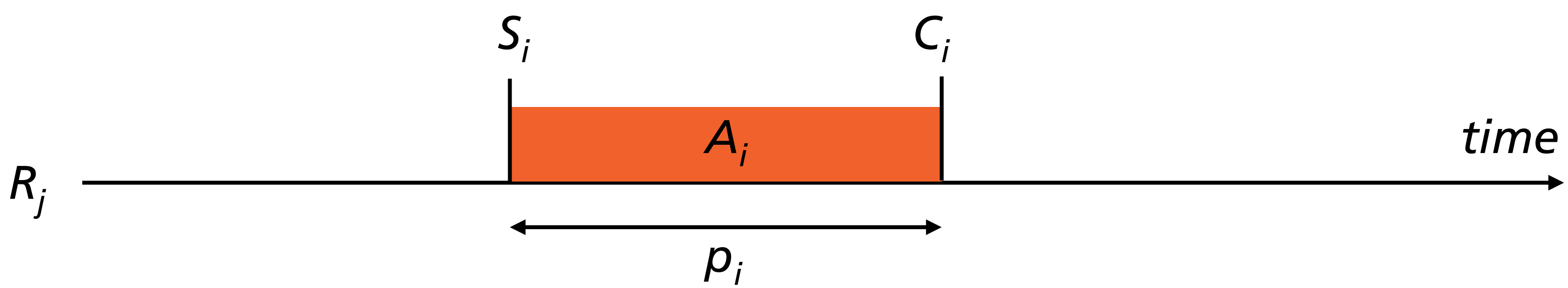
Scheduling – Activities and Resources

Activity A_i

- p_i – processing time
- S_i – start time (to be determined)
- C_i – completion time
- $C_i = S_i + p_i$

Resource R_j (can be machine, worker,…)



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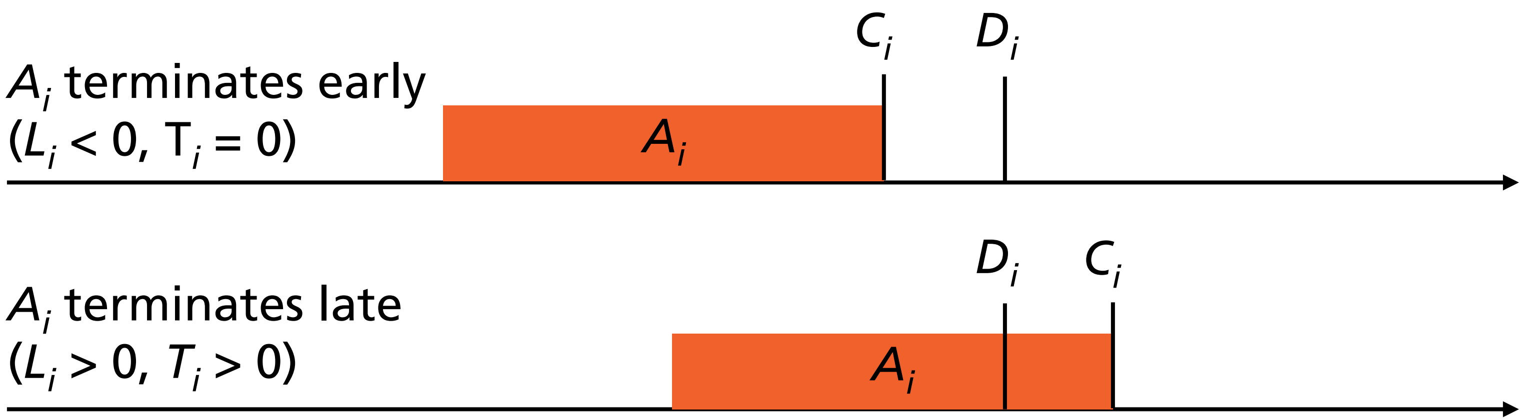
Scheduling – Lateness and Tardiness

Activity A_i

- D_i – due date

- $L_i = C_i - D_i$ lateness
- $T_i = \max(L_i, 0)$ tardiness



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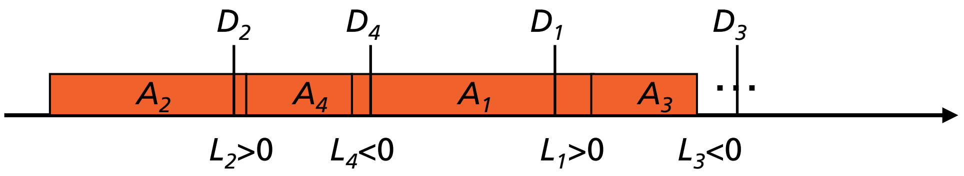
Scheduling – some examples for objective functions

Activities $A_1, A_2, \ldots A_n$



Task:
Schedule activities on a single resource (determine sequence) such that…

- $\max_i L_i$ is minimized → EDD rule is optimal („earliest due date")
- $\sum_i L_i$ is minimized → SPT rule is optimal („shortest processing time")
- $\sum_i T_i$ is minimized → problem is *NP*-hard, no rule exists

EDD rule: sort $A_1, A_2, \ldots A_n$ in non-decreasing order of due date

SPT rule: sort $A_1, A_2, \ldots A_n$ in non-decreasing order of processing time

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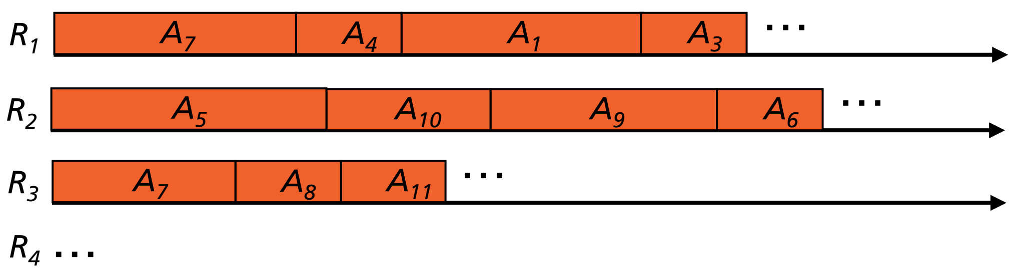
Multiple resources

Activities $A_1, A_2, \ldots A_n$

Resources $R_1, R_2, \ldots R_m$

Tasks:
(1) Assign activities to resources

(2) Schedule activities on each resource



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Example – Transport of Patients in Hospitals

Typical transport task:

- Bring patient Smith from ward to X-ray department for examination
- Performed by transport personnel

Model:

- Transport tasks – activities $A_1, A_2, \ldots A_n$
- Transport personnel – Resources $R_1, R_2, \ldots R_m$

Dispatcher:

- Assign tasks to personnel
- For each worker determine sequence of tasks

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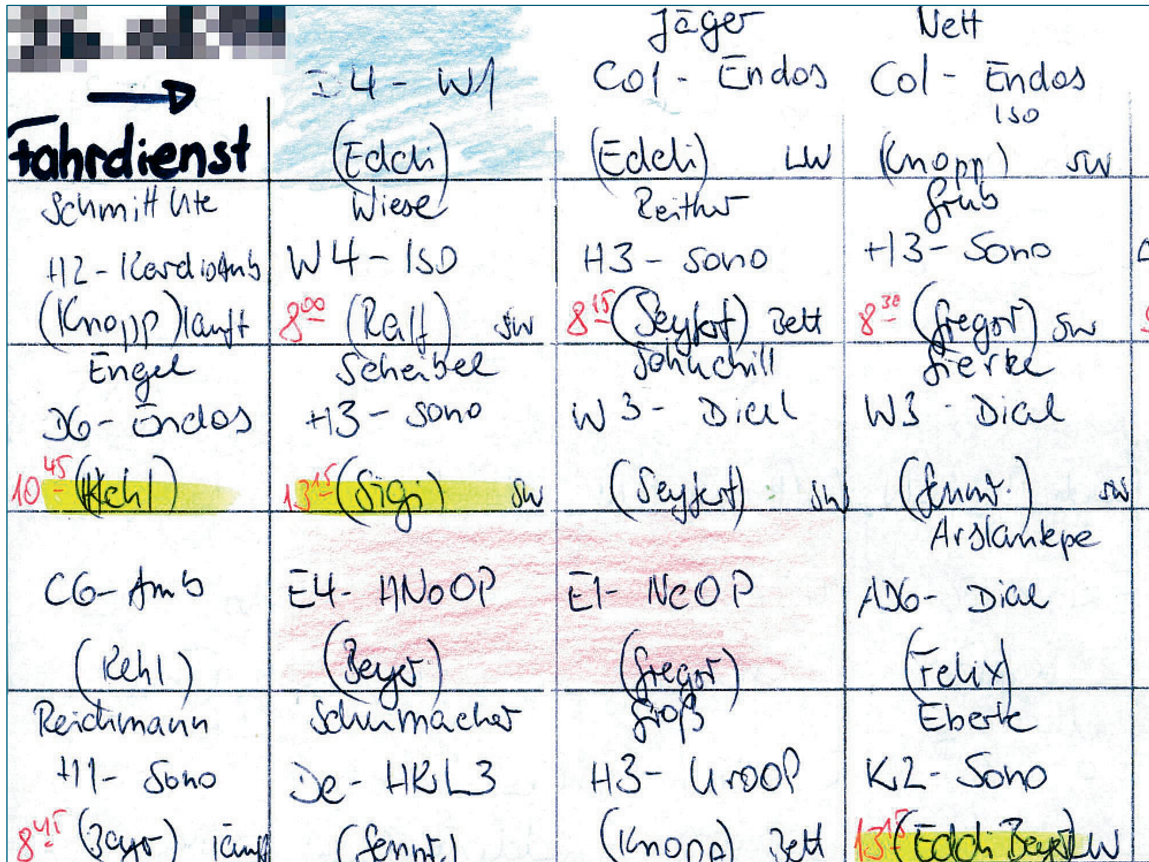
Dispatching of transport tasks with classical media

- telephone
- paper
- pencil
- …



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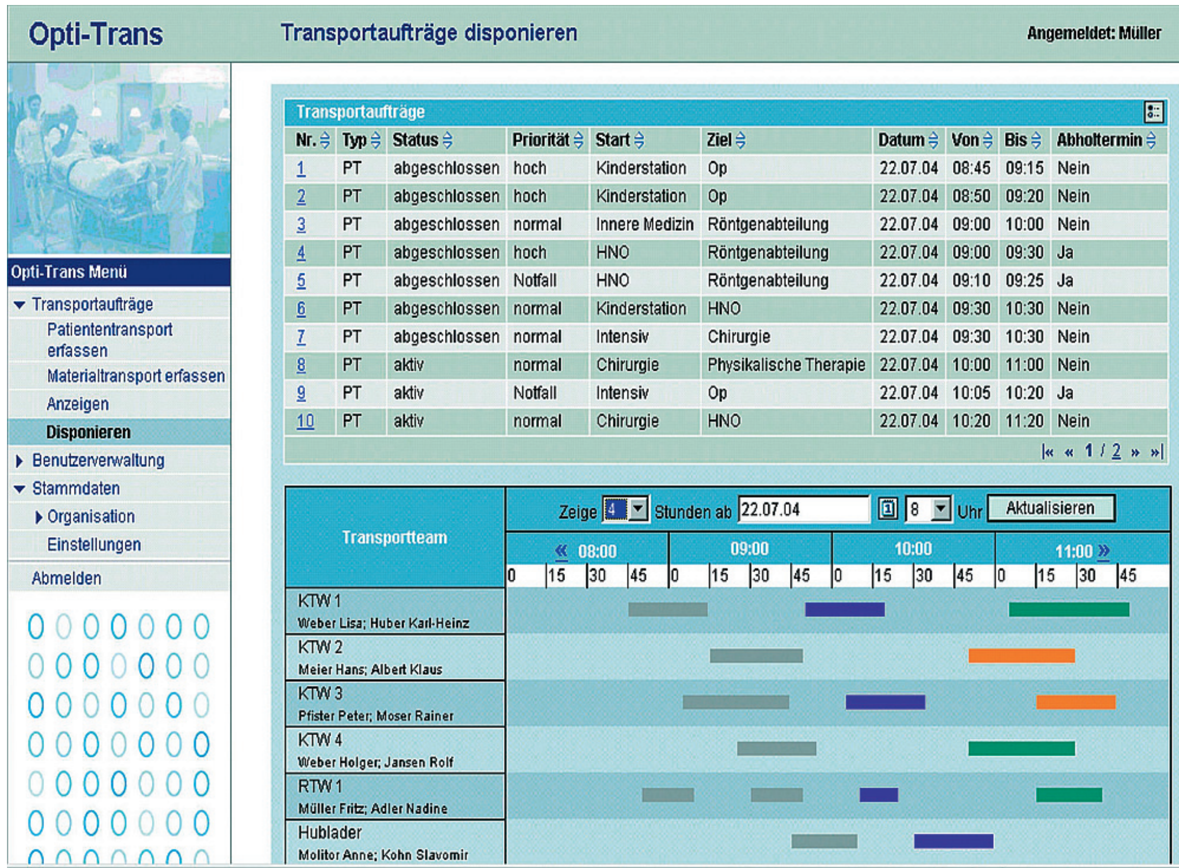
Dispatching of transport tasks with computer system

- Opti-TRANS®
- the whole process is software based
- dispatching algorithms



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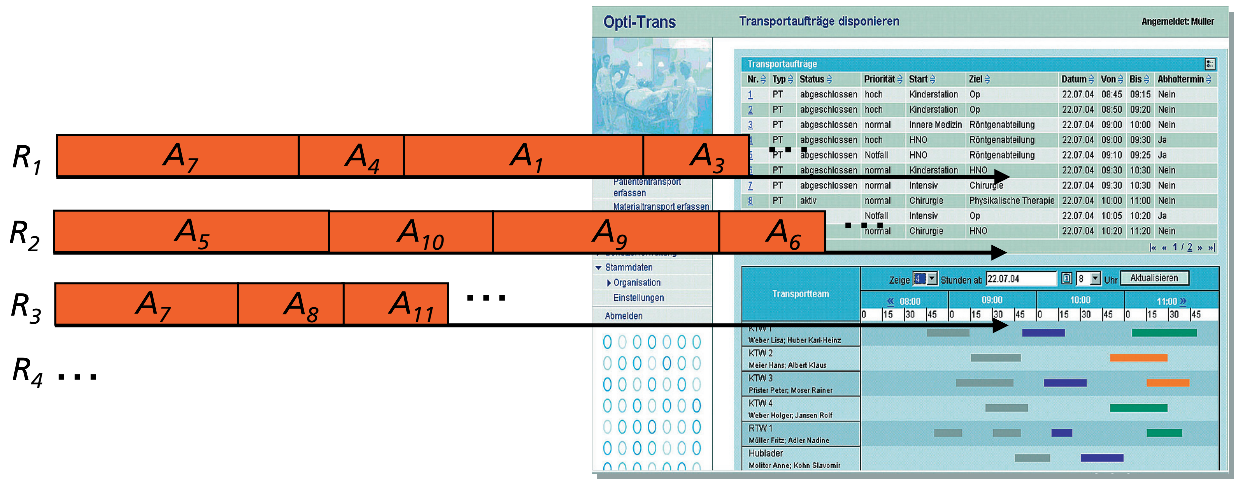
Objectives in Opti-TRANS® dispatching algorithm

- maximize timeliness (ideally lateness = 0 for all tasks)
- maximize resource utilization (avoid long ways between two tasks)
- balance workload on resources



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Balancing of workload for human resources

- very important for transport personnel
- appropriate measure for workload?
  - e.g. number of tasks
  - e.g. total processing time

- human resources are not always available (working times, breaks,…)
- workload measure has to be neutral to times of absence

- set of transport tasks changes dynamically – frequent re-planning is necessary
- workload measure has to be stable over re-plannings

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Modeling can be learned by doing, not by listening or reading.

Modeling is „metastrategic knowledge“
(see Elsbeth Stern: „Lernen“, Pädagogik 58(1)72006)

> *„Metastrategic knowledge emerges at best as a byproduct of the acquisition of content knowledge. Metastrategic knowledge is learnable, but only in exceptional cases direct teachable.“*

Find a good balance in teaching mathematics and exercising modeling. The more mathematics we know, the better are the models.

What should we learn from example 1:

- That there is not **one** model, that there might be a hierarchy of models
- how we get simpler models from complex models (f.e. by asymptotic analysis = „modeling“ in British tradition)
- that we may use complexer models to identify parameters in simpler models
- that models in order to be useful must be evaluated in a given time with given tools; therefore, efficient algorithms are very important too
- that models should be as simple as possibloe, but also as complex as necessary!

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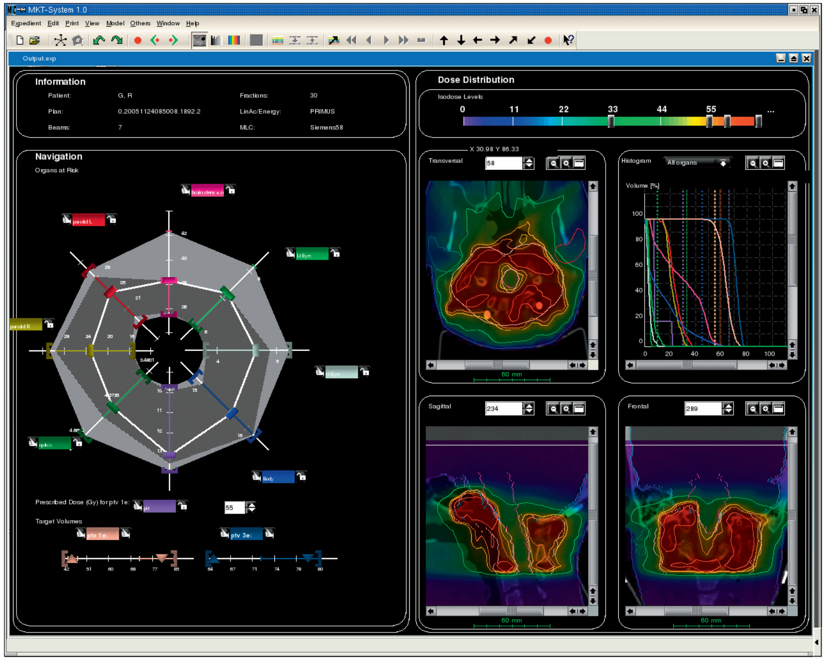
What should we learn from example 2:

- How scheduling processes may be modeled (even in schools)
- that one needs many personal contacts, when human decisions are involved
- that optimization problems very often have not only **one** objective function:

  Multicriteria Optimization



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Our experience in teaching modelling

- Modeling weeks for high school students (4 – 5 talented students per school) and some high school teachers.

  Since 25 years, 2 – 3 weeks per year.

- Export of Modeling activities to many European, Asian and South American countries.

- Personal visits in more than 300 companies all over the world with discussions about how to solve „nonstandard problems“ with mathematics.

Problem finding Competence

Modeling problems are everywhere, even „in the bakery around the corner".

Even in „walking in the rain":

- Is it worth while to run in rain?

Or by talking to colleagues of other disciplines:

- How do we identify tortoises?

Please do not „invent" problems!

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Modeling gives a „meaning“ to mathematics, creates a lot of enthusiasm in students

„Time passes by so quickly, since it is more interesting than lessons.“

„It was very interesting, since the problems were more complex and more realistic.”

„It was surprising that a problem has more than one solution.”

„It should be done more often.”

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Georg Christoph Lichtenberg (1742 – 1799)

“In order to find something, you have to know that it exists.”

➔ The teacher as a guide – not „eye in eye”, but „shoulder to shoulder”.

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