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Modelling energy forward prices

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Abstract— The main purpose of the paper is to present, how derivatives valuing methodology, known from financial and commodities markets, can be applied to the electricity market. We compare an application of three recent models. We start with the convenience yield approach, then we analyse the application of the interest rates methodology, proposed by Hinz et al. (2005). Finally, the last approach built by Bjerksund et al (2000) on direct modelling of the forward price dynamics is discussed. We also calibrate the theoretical models to the Nord Pool market data. The empirical analysis shows how these models can be used for evaluation of options prices. Moreover, data study gives an evidence of the seasonal term structure of the returns variance.

I. INTRODUCTION

As a consequence of the rapid changes in the electric energy market during the last two decades, market participants become exposed to the significant risk. Therefore, there is a necessity for development of effective risk management tools for the power market. The volume of the Nord Pool financial market, which exceeds the physical trade about five times, shows how important the financial electricity market became.

In a competitive power market electricity can be bought and sold at market price like any other commodity. Extreme price volatility, which can be even two orders of magnitude higher than for other commodities or financial instruments, has

forced producers and wholesale consumers to hedge not only against volume risk but also against price movements [3], [5], [10]. The most prominent of all models of modern finance - Black-Scholes model - could not be applied directly to electricity price. It does not allow for price spikes and mean-reversion [3], [10]. Moreover, due to a constant volatility assumption, it neglects seasonal time structure observed in the electricity market. Thus, other models are needed.

In this paper we analyse three recent models for valuation of electricity derivatives and calibrate the simplest common model to real market data from Nord Pool. The Nordic commodity market for electricity, known as Nord Pool, was established in 1992. It was the world's first international power exchange. Nord Pool offers the spot (physical) market - Elspot and the financial market - Eltermin, where power derivatives like forwards (up to five years ahead), futures, options and contracts for differences are traded. There are today hundreds of market participants from many countries active on Nord Pool. These include not only generators, suppliers/retailers, traders, large customers, but also financial institutions. The success of Nord Pool is well known, [10].

The present paper is structured as follows. In section 2 the MS model [7], the BRS model [1] and the HGVW [6] model are introduced. In order to facilitate the notation the models are called by initials of the authors. The main contribution - their comparison is presented in section 3. We focus on interpretation, assumptions on interest rates and option valuation approach. Next in Section 4 we demonstrate how to calibrate

the discussed models to the market data from Nord Pool. We perform, using data from Nordic Power Exchange, an empirical analysis for the special case when the forward dynamics and the option price are the same in all above models. This analysis is carried on, taking into account the special features of forward prices observed in the market. We consider not only increasing volatility when maturity approaches, like the authors of [1], [4], but also a weekly seasonal pattern.

II. MODELS FOR FORWARD PRICES

A. MS model

In classical financial theory a relationship between spot and forward prices is implied by the absence of arbitrage [8]. For commodities it means that the spot price should exceed the forward by the cost of carrying the physical commodity until the expiry date. This cost is determined by the physical storage costs and the interest lost by investing into a commodity [5], [10]. In practice this relationship rarely holds, due to limited ability to store a commodity and it does not make sense for non-storable commodities such as electricity. The notion of convenience yield, defined as a premium to a holder of physical commodity as opposed to a forward contract written on it, yields a solution to this problem. However, there is still a question, if we can consider the premium for holding a commodity according to non-storable commodities. Thus, instead of a reward for holding a commodity we will consider a premium for investing into a risky asset.

Regarding the definition of convenience yield δ_t , the relationship between spot S_t and forward $F(t, T)$ prices is given by

$$F(t, T) = S_t e^{(r_t - \delta_t)(T-t)},$$

where r_t is the risk-free interest rate.

Models with the spot price as the only stochastic factor are not appropriate for electricity markets because of different features of spot and forward prices. This remark comes from the fact that spot prices are driven by different stochastic factors (like actual weather conditions) than forward prices. Hence, we will focus on the MS model proposed by Miltersen and Schwartz [7]. The authors developed a model for commodity pricing with stochastic interest rates as well as stochastic convenience yield. For details and options pricing formula see [7].

B. BRS model

A standard approach to valuation of commodity derivatives is based on the relationship between spot and forward prices, given by a wealth of a replicating strategy. However, those methods can not be applied to power derivatives due to the non-storability of the electricity. The methodology proposed in the previous section circumvents this problem. However, it still requires modelling the relationship between spot and forward prices, which in case of electricity is not well recognised. As a response, a line of research has focused on modelling directly the dynamics of forward curves instead of spot prices. Such an approach does not require any assumptions about the relationship between spot and forward prices. Obviously, those methods are only appropriate for pricing derivatives written on forward contracts. An example of such an approach can be found in [1], [4].

In this section we will use some methods proposed by Bjerk Sund et al. [1]. However, we will choose another volatility function in line with our market observations.

Suppose, following [1], that the forward market is represented by a function $F(t, T)$, which denotes the forward price at date t of a MWh delivered at date $T \geq t$. Let $(W_t)_{t \in [0, T]}$ be a standard Brownian motion. Assume that the dynamics of the forward price is given by the following stochastic differential equation

$$dF(t, T) = F(t, T)(\mu(t, T)dt + \sigma_{BRS}(t, T)dW_t).$$

An empirical analysis of prices from Nord Pool shows that the volatility of forward price returns is a decreasing function of time to maturity $T-t$. Moreover, it indicates a dependence on the day of the week in which the price is quoted. Hence, we will consider the following form of the function $\sigma(\cdot, \cdot)$

$$\sigma_{BRS}(t, T) = \sigma e^{-\alpha(T-t)} + \beta(t), \quad (1)$$

where σ, α are constants and

$$\beta(t) = \begin{cases} a_1 & \text{if } t \text{ is Monday,} \\ a_2 & \text{if } t \text{ is Tuesday,} \\ \dots & \\ a_7 & \text{if } t \text{ is Sunday.} \end{cases}$$

Consider a European call option with strike price K and maturity date $\tau \in [0, T]$ written on a power forward contract maturing at T . Availability of the forward contracts completes the market. Using standard arguments, the market value of the option can be represented by the expected, with respect to the martingale measure, discounted future payoff. With a constant interest rate r and forward price being log-normal the call value at date $t \in [0, \tau]$ is given by the Black formula [8]

$$C_t = E[e^{-r(\tau-t)}(F(\tau, T) - K)^+ | \mathcal{F}_t] = e^{-r(\tau-t)}[F(t, T)N(d_+) - KN(d_-)],$$

where $N(\cdot)$ is a standard normal cumulative distribution function,

$$d_{\pm} = \frac{\ln(F(t, T)/K) \pm \frac{1}{2}\Sigma^2}{\Sigma},$$

and Σ is given by

$$\begin{aligned} \Sigma^2 &= \frac{\sigma^2}{2\alpha}(e^{-2\alpha(T-\tau)} - e^{-2\alpha(T-t)}) + \frac{2\sigma}{\alpha}(\beta(\tau)e^{-\alpha(T-\tau)} - \beta(t)e^{-\alpha(T-t)}) + \\ &+ \frac{2\sigma e^{-\alpha(T-t)-7}}{\alpha(1-e^{-7\alpha})} \sum_{k=0}^6 (\beta(t+6-k) - \beta(t+7-k)) e^{-\alpha k} \left(1 - e^{\frac{7\alpha}{7} \left[\frac{[\tau]-[t]+k}{7} \right]} \right) + \\ &+ \beta^2(t)([t]+1-t) + \beta^2(\tau)(\tau-[\tau]) + \sum_{k=1}^7 \beta^2([t]+k) \left[\frac{[\tau]-[t]-k+6}{7} \right]. \end{aligned}$$

C. HGVW model

Due to non-storability of electricity, forward power contracts with different times of delivery seem to be written on different commodities without any opportunity to transfer one commodity into another. Observe that a forward contract supplying 1 MWh at date T can be compared to a zero-coupon bond paying 1 cash unit at maturity moment T . Although the interest rate methodology seems to be appropriate for power contracts, there is an important difference which has to be considered. Observations of the forward electricity market show that forward prices fluctuate near the date of delivery T .

Hence, the volatility of power contracts must satisfy the condition $\lim_{t \rightarrow T} \sigma^2(t, T) > 0$. Obviously, this remark is not relevant for bonds, since bond prices converge to one when maturity approaches. The crucial point is to choose MWh as a "domestic currency", [6]. Then the price $p(t, T)$ at the moment t of a power forward contract maturing at date T satisfies $p(t, T) = 1$ so it can be modelled like a zero-coupon bond maturing at T , with a forward rate $f_{MWh}(t, T)$, see [8].

Suppose that there exists a discounting instrument N_t , which is a bank account in Euro paying a constant interest rate $r > 0$. It means that $e^{-rt} N_t$ is the reciprocal Euro-price at time t for electricity delivered at t . The MWh price of a forward contract is modelled like a zero-coupon bond in the Heath Jarrow Morton model [8]. For detailed model description and options prices derivation, see [2], [6].

III. COMPARISON

In this section we will compare three presented models. We will focus on three areas: interpretation, assumptions about interest rates and option valuation approach. Next, we will show, what are the relations between these models and finally we will point to each model's main advantages and disadvantages.

A. Interpretation

The convenience yield approach is a well known methodology for modelling commodity forward prices. However, in case of non-storable commodities, like electricity, the storage cost interpretation is not appropriate. Thus, we consider the reward for investing into a risky asset. A buyer of a forward contract agrees on how much he will pay for electricity at a specified future moment, so he avoids the risk of electricity price movements. Since investing in risky assets should generate higher profit than trading non-risky ones, there should be a risk premium related to the risky investments. In such a case convenience yield $\delta(t, T)$ can be interpreted as the expected rate of return above the risk-free interest rate seen from moment t for investing at date T .

In the MS model the forward price $F(t, T)$ is given by

$$dF(t, T) = F(t, T)(\sigma_S(u) + \int_u^T (\sigma_f(u, s) - \sigma_\delta(u, s))ds) dW_t,$$

where σ_S , σ_f , σ_δ denote volatility of a spot price, a forward interest rate and convenience yield respectively. Moreover,

$$|\sigma_S(t) + \int_t^T (\sigma_f(t, s) - \sigma_\delta(t, s))ds| \rightarrow \sigma_S(T) \quad \text{as } t \rightarrow T.$$

Thus, the volatility of forward prices converges to the spot one, while approaching maturity. Since market observations indicate higher volatility on the spot market than on the forward one, the MS model gives an interpretation of increasing volatility of forward prices before maturity.

We should, however, remember that this model requires modelling the forward-spot relationship. Moreover, with assumed dynamics of modelled processes, the forward and spot prices have similar features, what is not consistent with market observations.

The reason why Hinz et al. [6] use an interest rates model on the forward electricity market is that a forward contract

supplying 1MWh during time interval $[T, T+\Delta T]$ is similar to the zero-coupon bond paying 1 cash unit at maturity date T . The choice of MWh as the domestic currency causes that forward contracts converge to one when maturity approaches, just like zero-coupon bonds. However, there is still an important difference between forward contracts and bonds, which is not taken into account in this approach. We should remember that at the moment of entering a forward contract there are no cash flows, while a bond has face value which is paid at the moment of purchase. Thus, the application of the HJM model proposed in [6] is questionable.

Moreover, there are some problems with interpretation relevant to real-market conditions. First is how can we interpret the process N_t . The real-market asset that describes the Euro/MWh relation is the spot price. However, due to non-storability of electricity it can not play the role of a savings account. Furthermore, the discounting instru-

ment $B_t = \exp(\int_0^t f_{MWh}(s, s)ds)$ should reflect the time value of held 1 MWh so its application to non-storable commodities seems to be inappropriate.

On the other hand, disregarding the suitability of the HGVM model for the power market, there are some analogies to the convenience yield approach. The Euro price $E(t, T)$ at date t of a power forward contract is given by

$$E(t, T) = \frac{p(t, T)}{N_t} e^{rt} = \frac{1}{N_t} \exp(rt - \int_t^T f_{MWh}(t, s)ds),$$

where N_t is an analogy to the electricity spot price but with MWh as a domestic currency (MWh/Euro) and $p(t, T)$ being a MWh bond is an analogy to the storage costs during period $[t, T]$.

Moreover, similarly as the MS model, the HGVM formulation also gives an interpretation of the increasing volatility of forward electricity contracts near maturity, since for forward rate volatility $\sigma_{f_{MWh}}(t, T)$ we obtain

$$|\int_t^T \sigma_{f_{MWh}}(t, u)du - v_t| \rightarrow v_T \quad \text{as } t \rightarrow T,$$

and v_t is the volatility of the process N_t , which can be interpreted as an analogy to the spot price.

The BRS model assumes that the forward price dynamics is given ad hoc, so the questions about interpretation stay unexplained. However, the model does not require any assumptions about forward-spot relationship.

B. Interest rates

The MS model is the most general concerning interest rates. The authors consider a Gaussian stochastic term structure described by the HJM formulation. By contrast, in the HGVM model we have two markets with different currencies and different interest rates - the HJM stochastic term structure with savings account as a numeraire on the MWh-market and a constant interest rate on the Euro-market. The simplest case with a constant interest rate is considered in the BRS model.

C. Option valuation

In each described model an option is priced as an expected discounted future payoff. However, the choice of measure is different. In both, the BRS and the MS models, the forward

price is a martingale [8] with respect to the chosen measure. It comes from the fact that there are no initial investments required by forward contracts transactions [8]. In contrast, in the HG VW model the discounted (with respect to e^{rt} on the MWh-market and B_t on the Euro-one) forward price is assumed to be a martingale.

An important part of option valuation is finding hedging opportunities. While the assumption of availability of forward contracts for any future date T gives a replicating strategy for the option in the MS and the BRS models, the construction of the MWh-market in the HG VW model leaves completeness of the market unsolved.

D. Special case

Assuming a constant interest rate in the MS model we obtain the following results:

- the forward price dynamics is described by

$$dF(t, T) = F(t, T)(\sigma_S(t) - \int_t^T \sigma_\delta(t, s) ds) dW_t,$$

- the option price formula simplifies to

$$C_t = e^{-r(\tau-t)} \left[F(t, T) N(d_+) - KN(d_-) \right]$$

$$d_\pm = \frac{\ln(F(t, T)/K) \pm \frac{1}{2} \Sigma^2}{\Sigma},$$

$$\Sigma^2 = \text{Var} \left(\ln \frac{F(\tau, T)}{F(t, T)} \right).$$

Thus, the BRS model can be interpreted as a special case of the MS model with a constant interest rate and volatility fulfilling

$$\sigma_S(t) - \int_t^T \sigma_\delta(t, s) ds = \sigma_{BRS}(t, T).$$

To compare the HG VW model with the other we will focus on the dynamics of the forward price and the way the European option is priced. We have checked that reformulating the model for the Euro currency we get the standard option price formula. Moreover, it can be showed that the forward price in Euro is described by the following stochastic integral

$$E(u, T) = E(s, T) \exp \left(r(u-s) + \int_s^u \beta_x dW_x - \frac{1}{2} \int_s^u \beta_x^2 dx \right),$$

with $\beta_t = s(t, T) - v_t$, where $s(t, T)$ is the volatility of the forward price in MWh. Thus, all described models are equivalent only if the interest rate $r = 0$ and volatility satisfies

$$\sigma_S(t) - \int_t^T \sigma_\delta(t, s) ds = \sigma_{BRS}(t, T) = s(t, T) - v_t.$$

In Table 1 we have pointed out main advantages and disadvantages of the models.

TABLE 1
ADVANTAGES AND DISADVANTAGES

Model	Advantages	Disadvantages
MS	stochastic term structure of interest rates	forward-spot relationship
	realistic interpretation	
	existence of the options replicating strategy	
	explanation of increasing volatility	
BRS	simple analysis	constant interest rate
	no forward-spot relationship requirements	no interpretation
	existence of the options replicating strategy	
HG VW	application of interest rates methodology	non-realistic interpretation
	explanation of increasing volatility	differences between bonds and forward contracts

IV. CASE STUDY: NORD POOL DATA ANALYSIS

In this section following [2] we will show how to calibrate the presented theoretical models to real market data. Since the forward price dynamics and the options prices are the same in all models if we assume interest rate $r = 0$ and volatility from the BRS model, we will consider only this special case of the described models.

The volatility $\sigma(t, T)$ can be estimated as

$$\hat{\sigma}(t, T) = \sqrt{\text{Var} R(t, T) \frac{1}{\Delta t}}$$

and the drift $\mu(t, T)$ as

$$\hat{\mu}(t, T) = E \left(\frac{R(t, T)}{\Delta t} \right) + \frac{1}{2} \hat{\sigma}^2(t, T),$$

where $R(t, T)$ is the logarithmic return of the forward price. Since we consider daily closing prices and time unit equal to one day, $\Delta t = 1$.

Because there is no trade on Nord Pool financial market during weekends, we will consider only five business days of the week. In order to cope with the lack of data during holidays we will use the similar day approach proposed in [10]. The missing values will be substituted by the average of the previous and the following prices from the same day of the week.

Characteristics of instruments traded on Nord Pool financial market have changed within the time of market's activity. The month contracts were introduced at the end of year 2003 and the quarter contracts were first launched in year 2004 with settlement in year 2006. The only contracts that have been traded from the beginning of the market until now are week and year ones. However, in case of year contracts there is only one contract for each year. Therefore we will focus on the week contracts, for which there is most quotation data. In order to obtain time dependence in volatility structure we will analyse closing daily prices from years 1996-2002, because from year 2003 the trading time of the week contracts was shortened to six weeks only.

We assume that the contract price is independent from the length of delivery period. However, with assumed volatility's form contracts with one length of delivery period can be easily scaled to the contracts with other duration of delivery.

In Fig. 1 we have plotted the estimated volatility of logarithmic returns of the week forward prices. In order to illustrate, how the volatility varies depending on the day of the week, each day is plotted with different colour.

Fig. 1. Volatility's dependence on time to maturity estimated from week forward prices in years 1996-2002. Data from Nord Pool financial market

Fig. 1 shows the increasing volatility of the forward prices when maturity approaches and also gives an evidence of the relationship between volatility and day of the week. It should be mentioned that much higher Monday volatility is a consequence of the weekend uncertainty, which we do not directly consider due to the lack of data. However, even with the assumption of equal volatility on Saturday, Sunday and Monday the week pattern is still visible.

We will fit estimated volatility to the function defined in (1). However, we will assume Monday to Friday seasonal pattern. The values of parameters α , σ , a_i are found by minimising the squared error (using Matlab simplex search routine), Table 2.

TABLE 2
VOLATILITY FUNCTION PARAMETERS

	σ	α	a_1	a_2	a_3	a_4	a_5
$\beta(t)$	0.0182	0.0334	0.0351	0.0219	0.0216	0.0229	0.0171

In Fig. 2 we have plotted the estimated drift of the logarithmic returns of the week forward prices.

Fig. 2. Drift dependence on time to maturity estimated from week forward prices in years 1996-2002. Data from Nord Pool financial market.

The estimated values oscillate around zero, so we will assume that $\mu(t, T) = 0$ for all $t \in [0, T]$. This means that the forward price is a martingale under the real market measure.

First, we will check how the option price depends on the maturities of the option and the underlying contract. For this purpose we will find the prices on 02/01/07 of the options written on the month contracts with delivery within next six months. The prices will be evaluated for options with maturities on each day till the end of the underlying contract trading period. In order to check how the underlying contract delivery period influences the option price we will use one strike price for each option calculated as the arithmetic mean of the six considered contracts prices on 02/01/07. Results have been plotted in Fig. 3.

Fig. 3. The option prices with strike price set as an arithmetic mean of the considered six underlying contracts prices on 02/01/07. Data from Nord Pool financial market.

Comparing options with equal distance between option and contract maturity, we notice that the option price is the highest for delivery in February and the lowest for delivery in July. This result is a consequence of the seasonal effects in electricity prices. The cost of electricity is the highest during winter and the lowest during summer. Thus, the prices of options with the same strike prices are higher for the winter months than for the summer ones.

Now, we will compare the model results with the market prices. We will consider the option traded on Nord Pool financial market specified in Table 3.

TABLE 3
DETAILS OF THE CONSIDERED OPTION, QUOTED ON NORD POOL

Option type	Underlying contract	Strike price	Maturity of the option	Maturity of the contract	Start of delivery
call	2Q-07	23	15.03.07	30.03.07	01.04.07

We will find the option price in each day from 09/01/07 to 14/03/07 and compare it with the market price of the considered option quoted in those days. We should however remember that due to the very low liquidity of the options trade on Nord Pool financial market this comparison can only be treated as an illustration of some trends. In Fig. 4 we have plotted the model and the market price of the considered option.

Fig. 4. Price of the considered option from 09/01/07 to 14/03/07. Data from Nord Pool financial market.

We observe that although there are some differences between the market and the model prices, the results are comparable.

V. CONCLUSION

The paper shows possible applications of the classical options pricing theory to the power market. The empirical analysis illustrates how these models can be used for evaluation of options prices. Moreover, data study gives an evidence of the seasonal term structure of the returns variance and enables better understanding of the forward electricity market.

VI. REFERENCES

- [1] P. Bjerksund, H. Rasmussen, G. Stensland, *Valuation and risk management in the Nordic electricity market*, Working paper, Institute of Finance and Risk Management Science, Norwegian School of Economics and Business Administration, **2000**.
- [2] E. Broszkiewicz-Suwaj, A. Weron, *Calibration of the multi-factor HJM model for energy market*, Acta Phys. Polon. B, 37(**2006**), 1001-1012.
- [3] D. W. Bunn (ed.), *Modelling Prices in Competitive Electricity Markets*, Wiley, Chichester, **2004**.
- [4] L. Clewlow, C. Strickland, *Valuing energy options in a one factor model fitted to forward prices*, Research Paper Series 10, Quantitative Finance Research Centre, University of Technology, Sydney, **1999**.
- [5] H. Geman, *Commodities and Commodity Derivatives. Modelling and Pricing for Agriculturals, Metals and Energy*, Wiley, Chichester, **2005**.
- [6] J. Hinz, L. Grafenstein, M. Verschuere, M. Wilhelm, *Pricing electricity risk by interest rate methods*, Quant. Fin., 5(**2005**), 49-60.
- [7] R. Miltersen, E.S. Schwartz, *Pricing of options on commodity futures with stochastic term structures of convenience yields and interest rates*, J. Financ. Quant. Anal., 33(**1998**), 33-59.
- [8] M. Musiela, M. Rutkowski, *Martingale Methods in Financial Modelling*, Springer, Berlin, **1997**.
- [9] I. Vehvilainen, *Basics of electricity derivative pricing in competitive markets*, Appl. Math. Fin., 9(**2002**), 45-60.
- [10] R. Weron, *Modelling and Forecasting Electricity Loads and Prices: A Statistical Approach*, Wiley, Chichester, **2006**.
- [11] www.nordpool.com

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