A PROBABILITY MODEL FOR THE ELECTRICITY PRICE DURATION CURVE

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Introduction

Polish energy market is still evolving, unfortunately changes on the market are going very slowly. We hope that optimal decision methods used on European markets could be useful on the polish market too. On European markets, where market mechanisms are fully developed, optimal decisions are highly dependent on market electricity prices. The basic risk management functions of market participants also depend on the knowledge of the future behaviour of market electricity prices.

On the other hand, the electricity price over the long term is a quantity that depends on physical factors such as production cost, load, generation reliability, unit commitment, and transmission constraints. It also depends on economic factors such as strategic bidding and load elasticity.

Introduction

We present a stochastic system-based model for electricity prices, taking into account the randomness associated with load and generator outages. The quantity that we model is the "price duration curve" (PDC). The PDC notation is well known among industry analysts in the USA and is widely used in decision making:

$$\bar{F}_p(y) = \frac{1}{T}\sum_{t=1}^{T} Pr(p(t) > y),$$

where T is planning horizon, $p(t)$ is price of electricity in time t. The electricity price $p(t)$ is a stochastic quantity.

The probabilistic interpretation of the PDC is following: PDC measures the average probability that the hourly price will exceed a quantity y, the average being taken over all hours belonging to the planning time interval.

Modelling the stochastic supply

We assume that there are n firms on the market. The firm number i has k_i generators which are participants on the market. PDC will be computed with respect to these generators over a time interval $[0, T]$. We also make the following additional assumption:

- If i-th firm is a participant in the market at time t, then it starts all its generators.
- We consider two models of $p(t)$: Bertrand model and extended Rudkevich model (oligopoly).
- Under the Bertrand model, the generators bid their marginal costs so that the price equals the marginal cost of the most expensive generators used for dispatch. The dispatch is done according to a predetermined loading order based on an increasing order of the marginal cost of each unit. This order

depends on the instantaneous system load and the availability of the generating units.

- In both of these models, unit commitment constraints are ignored.
- The market consists $k_1 + k_2 + \cdots + k_n$ generators. Each generator of i-th firm has a capacity c_i (MW), forced outage rate q_i and marginal energy cost d_i. The availability of the generator is characterized by the probability $p_i = 1 - q_i$.
- We assume that a firm's price bid is bounded on the above by the quantity D.

Bertrand model

We denote the last unit in the cumulative loading order used to meet the load or the marginal unit at hour t by $J(t)$. This value is a random variable because it depends on the load at time t and on the operating states (”on” or ”off”) of each generating unit prior to it in the loading order. Under the assumption of perfect competition, the market price at time t is the marginal cost of the unit $J(t)$:

$$p(t) = d_{J(t)}.$$

After calculating we obtain the following formula for PDC:

$$\bar{F}_p(y) = 1 - \frac{1}{T} \sum_{t=1}^{T} G_{h(y)}(X_{h(y)}; t),$$

where $h(y) = j$ if $d_j \leq y < d_{j+1}$, $G_j(x; t)$ is the distribution function of the load at time t after the first j firms have been loaded their generators and X_i is the cumulative capacity of the generators for first i firm.

Extended Rudkevich model

Under the assumption that the price equals the price cap D, when the load reaches the maximum system capacity, and such a load requires the utilization of the last unit of the system the expression for the market price for a symmetric oligopoly is given by:

$$p(t) = d_{J(t)} + \sum_{i=J(t)}^{n} (d_{i+i} - d_i) \left(\frac{L(t)}{X_i} \right)^{n-1},$$

where d_i is the marginal cost of i-th firm, $J(t)$ is the last unit in the cumulative loading order used to meet the load or the marginal unit at hour t, $L(t)$ is the load at hour t and X_i is the cumulative capacity of the generators for first i firm.

In this case the PDC is given by:

$$\bar{F}_p(y) = \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{n} r_j(a_j(y);t) \left[G_j(X_j;t) - G_{j-1}(X_{j-1};t) \right],$$

where

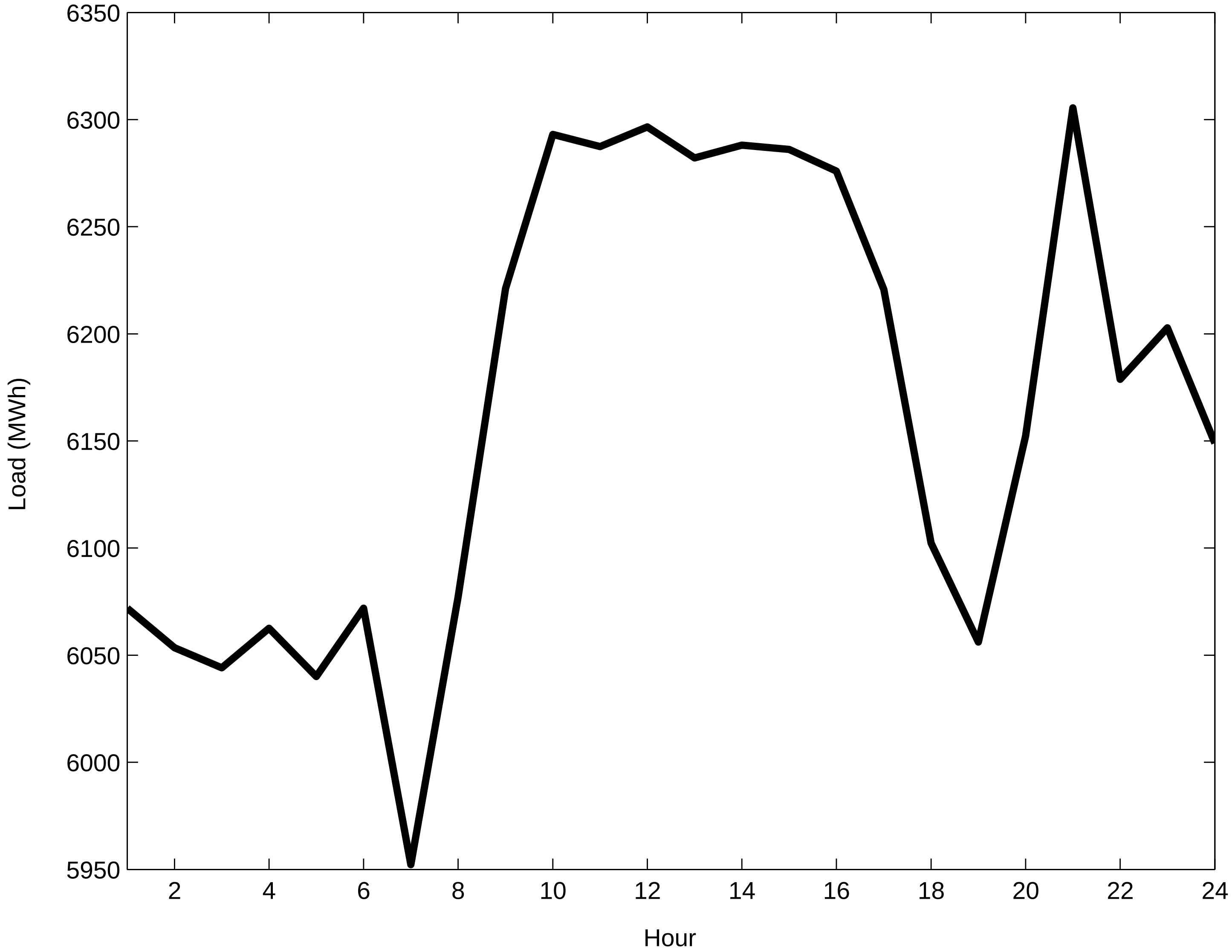
$$a_j(y) = [\max(0, y - d_j)/(\sum_{i=j}^{n} (d_{i+1} - d_i)/(X_i)^{n-1})]^{\frac{1}{n-1}},$$

$$r_j(a;t) = Pr(\bar{L}_{j-1}(t) > a | X_{j-1} \leq \bar{L}_{j-1}(t) < X_j),$$

$\bar{L}_j(t)$ is an equivalent load at time t considering first j firms.

Experimental results

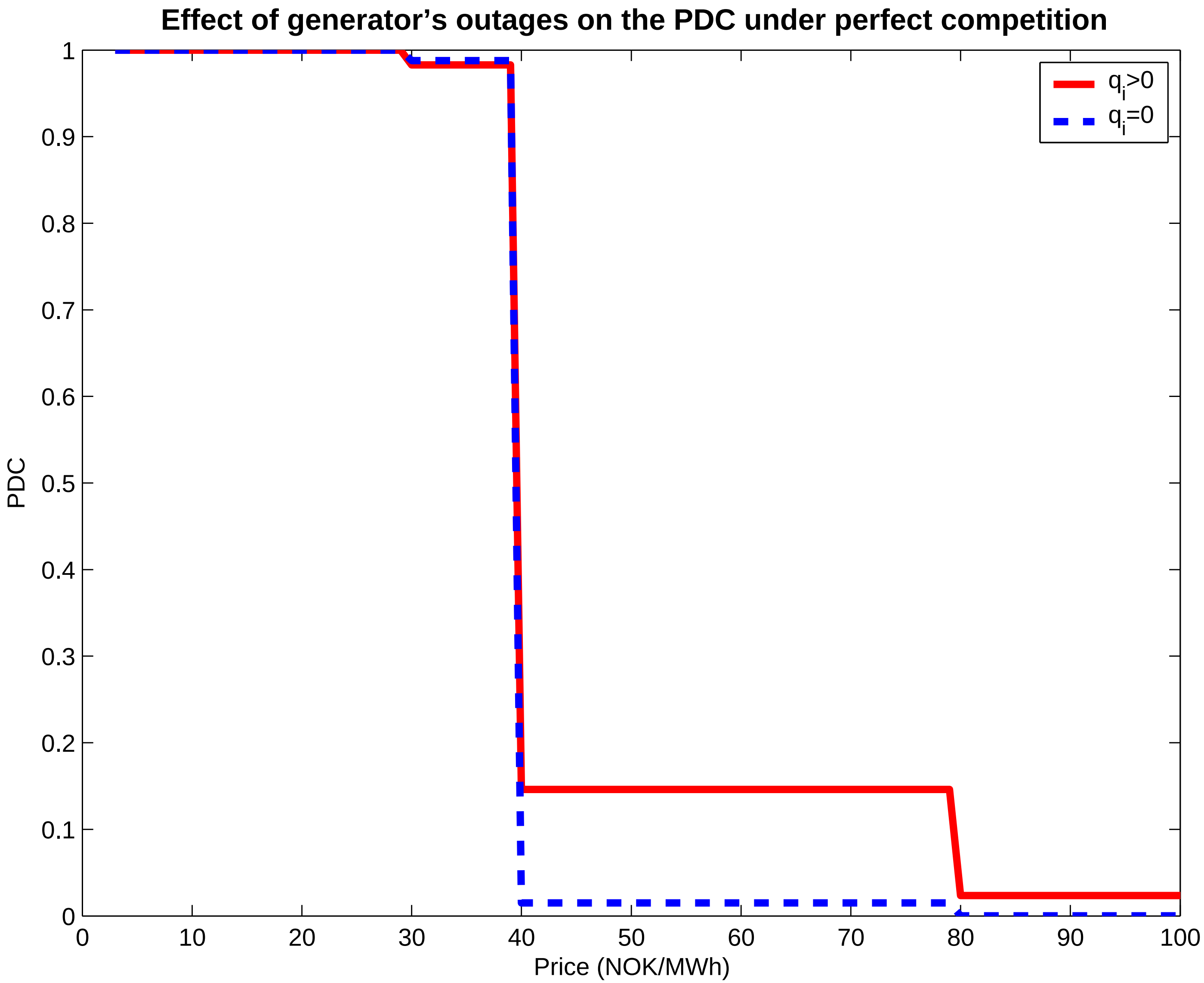
We consider volume power trading data from the Nord Pool Power Exchange. On Figure 1 we illustrate the value of hourly mean of the load for period 07.01.1999 - 07.03.1999.

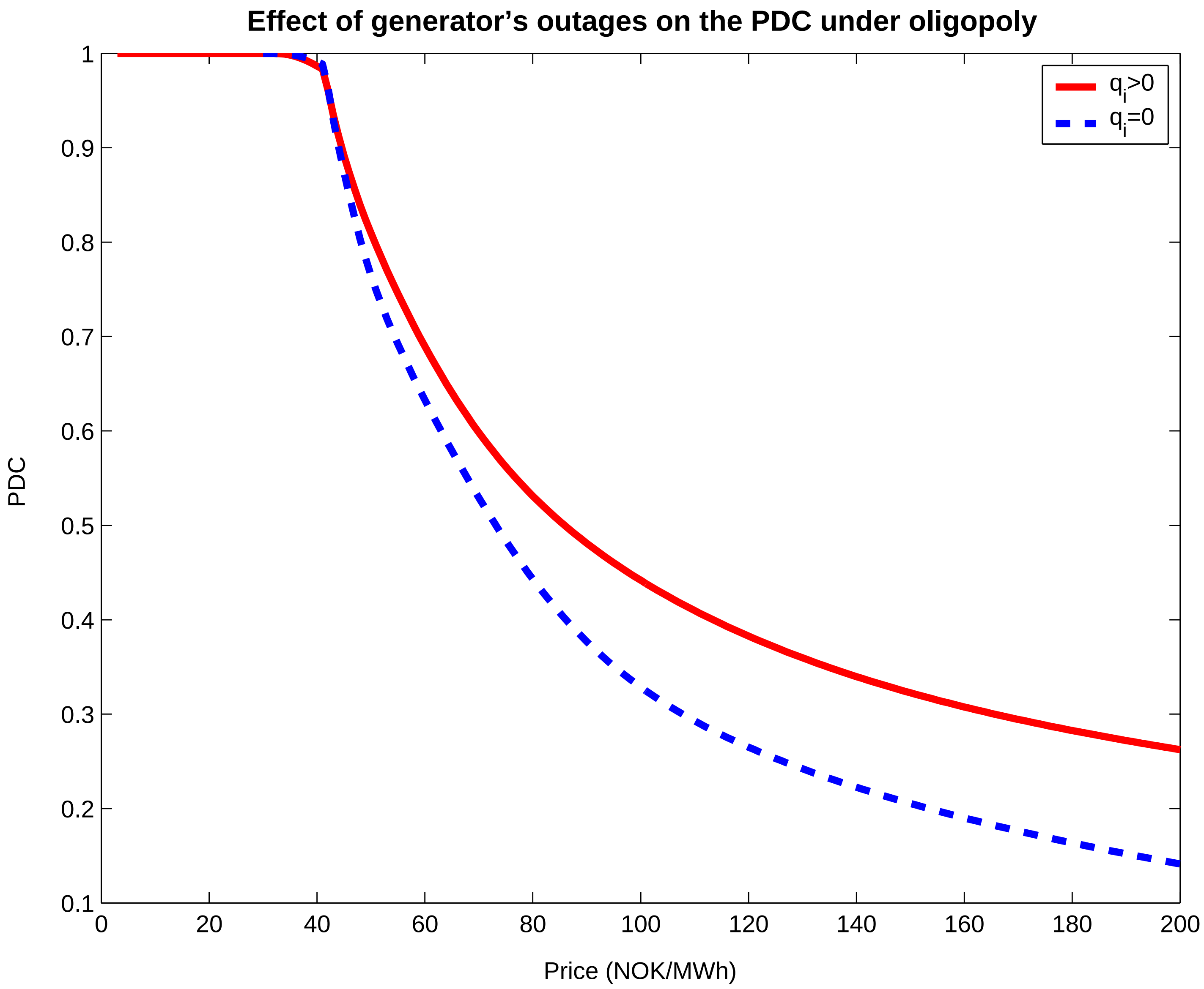

6350
6300
6250
6200
6150
6100
6050
6000
5950
Load (MWh)
2
4
6
8
10
12
14
16
18
20
22
24
Hour

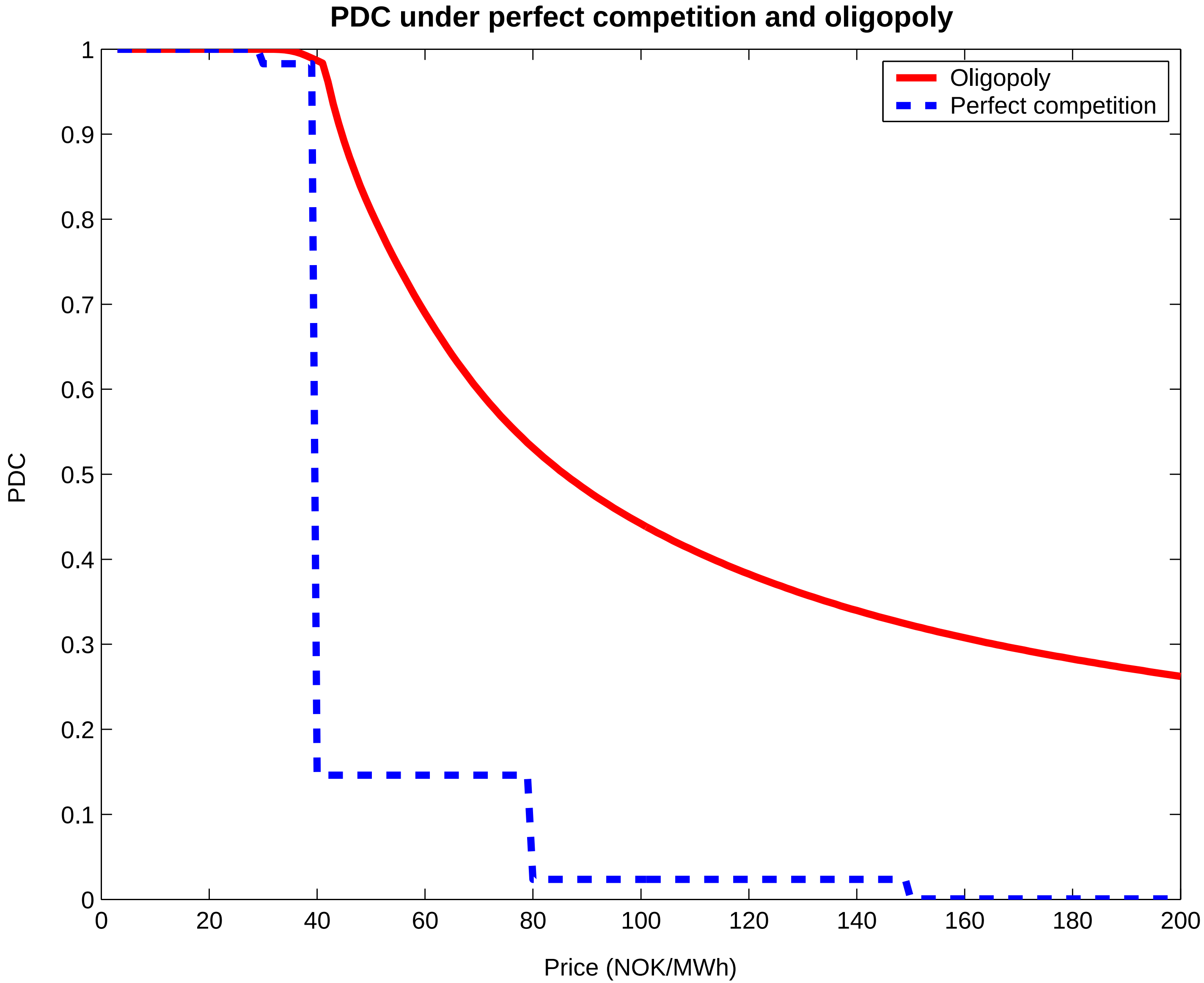
We assume there is hypothetical supply system with 6 firms with the following set of generators and c_i, d_i and q_i parameters:

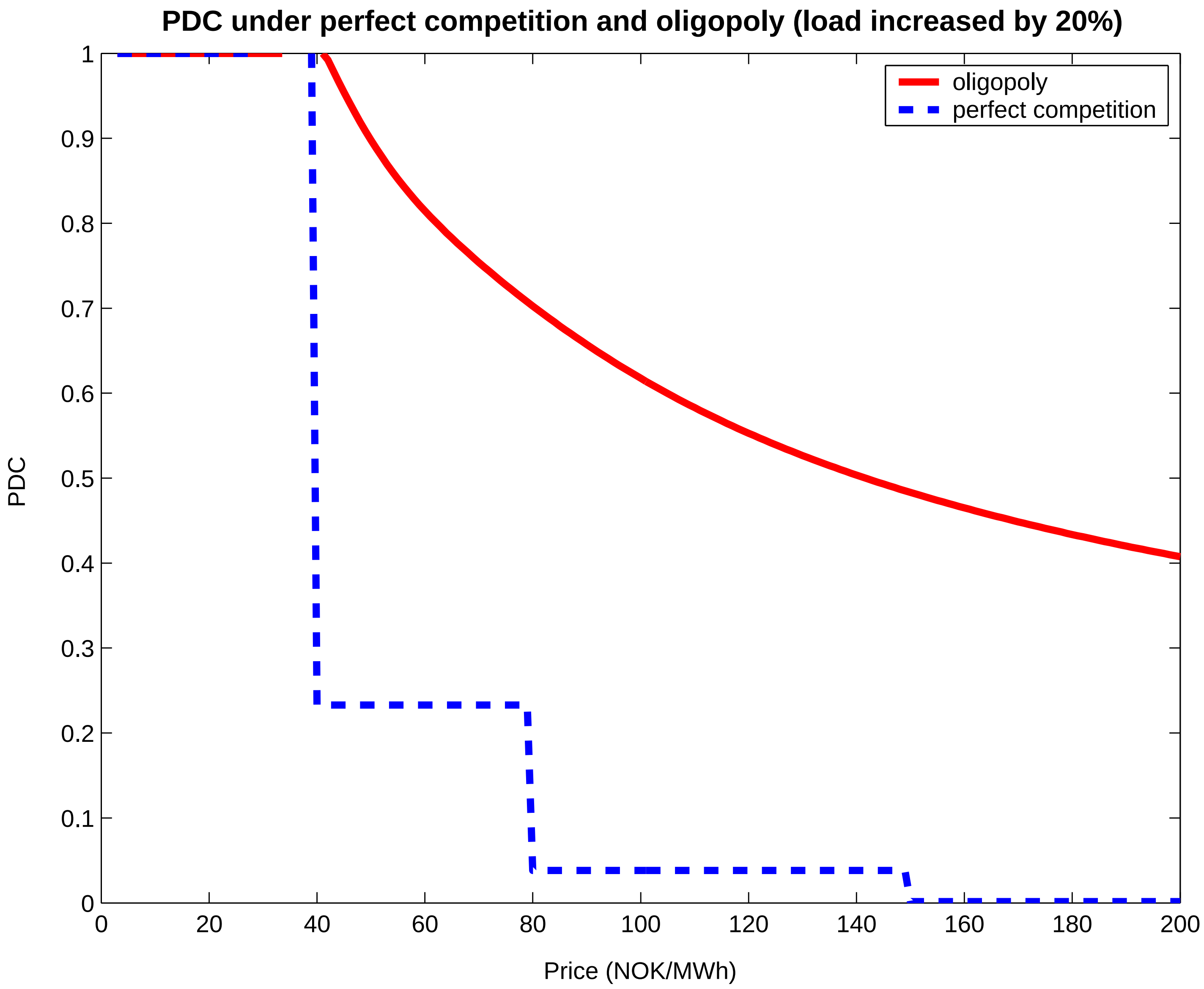
Firm	Number of generators	c_i	d_i	q_i
1	10	200	30	0.12
2	5	338	40	0.12
3	8	280	80	0.08
4	10	355	150	0.04
5	8	250	220	0.04
6	6	200	290	0.04

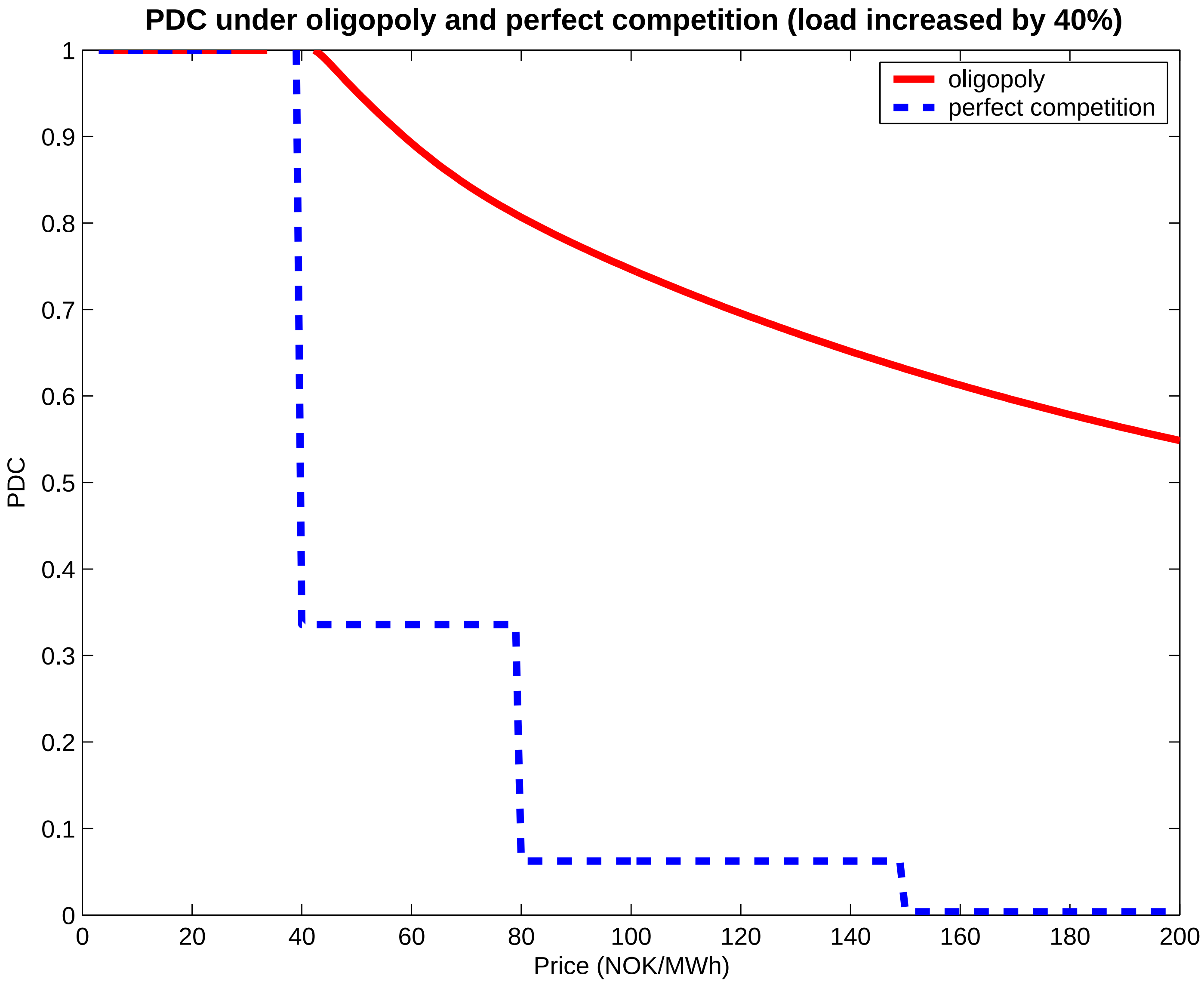
This power system has a total capacity of 12680 MW. Moreover we assume that the time horizon $T = 24$ (hours).
We first compute the PDC under perfect competition (Bertrand model) and show the behavior of that function under the assumption $q_i > 0$ and $q_i = 0$. Next we compare the PDC for two considered models and examine "price duration curve" for oligopoly (extended Rutkevich model) in two cases: when $q_i > 0$ and $q_i = 0$. In the last part we present the behavior of PDC, when the load increases by 20% and 40%.


Effect of generator's outages on the PDC under perfect competition
PDC
Price (NOK/MWh)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80
90
100
q_i>0
q_i=0


Effect of generator's outages on the PDC under oligopoly
$q_i>0$
$q_i=0$
PDC
Price (NOK/MWh)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
40
60
80
100
120
140
160
180
200


PDC under perfect competition and oligopoly
Oligopoly
Perfect competition
PDC
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
Price (NOK/MWh)


PDC under perfect competition and oligopoly (load increased by 20%)
oligopoly
perfect competition
PDC
Price (NOK/MWh)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200


PDC under oligopoly and perfect competition (load increased by 40%)
oligopoly
perfect competition
PDC
Price (NOK/MWh)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
20
40
60
80
100
120
140
160
180
200

Conclusions

- The PDC gives the mean of probability in period T that chosen price is higher than any price within the range of the anticipated prices.
- We analyzed the PDC with the forced outages and without them. The effects of outages were shown in case of perfect competition and in case of oligopoly. In both cases we observed higher PDC in case of forced outages ($q_i > 0$).
- We present the effect of market structure on the PDC. Obtaining results show that PDC in case of perfectly competitive market is lower than in oligopoly.
- Created computer programs, used to calculate the PDC, gave the possibility of observing how increasing load influences on the PDC.