FINANCIAL ENGINEERING AS A TOOL OF POWER PRODUCERS’ STRATEGIES OPTIMIZATION

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INTRODUCTION

- We consider the forward contracts and european call options for power delivery as the strategies of security of the players' profits on the electricity market.
- We show how the derivatives can affect the profit.
- We propose a linear asymmetric supply function equilibrium to develop firms' optimal bidding strategies considering forward contracts and european call options.
- We extend the methodology proposed in [4], where as profit security were considered only forward contracts for power delivery.

THE FORWARD CONTRACTS

- A forward contract is an agreement between two sides to buy or sell an asset at a pre-agreed future point in time. It is used to control and hedge risk, for example currency exposure risk.
- One party agrees to buy, the other to sell, for a forward price agreed in advance. In a forward transaction, no actual cash changes hands. If the transaction is collated, exchange of margin will take place according to a pre-agreed rule or schedule. Otherwise no asset of any kind actually changes hands, until the maturity of the contract.

THE FORWARD CONTRACTS

- The forward price of such a contract is commonly contrasted with the spot price, which is the price at which the asset changes hands.The difference between the spot and the forward price is the forward premium or forward discount.
- The forward contract pertains to future total value of 1 MWh of electricity at each hour, calculated at final settlement rate against the total value of 1 MWh of electricity in the same period. During quotation period the market players may change their positions by making subsequent purchase and sale transactions of the same contract.

THE OPTIONS

- An option is a contract, or a provision of a contract, that gives one party (the option holder) the right, but not the obligation, to perform a specified transaction with another party according to specified terms.
- Option contracts are a form of derivative instrument. Stand-alone options trade on exchanges or OTC. They are linked to a variety of underlies. Most exchange-traded options have stocks or futures as underlies. OTC options have a greater variety of underlies, including bonds, currencies, physical commodities, swaps, or baskets of assets. They take many forms. The two most common are: call options and put options.

NOMENCLATURE

- t_0 - the starting time point, when the players buy the forward contracts or the options
- $[T,U]$ - time period, when the forward contract or the option is realised (i.e. The power delivery)

$$t_0 < T < U$$

- p_t - the market price
- profit function for firm *j* at time *t* is denoted by π_{jt}

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FORMULATION OF THE SUPPLY FUNCTION MODEL

- We assume there are *m* power producers on the power market. The player *j* gives his bid q_j , that is a function of p_t and is dependent on two parameters α_j and β_j.
- The bid curve is given by:

$$q_j(p_t) = \beta_j(p_t - \alpha_j)$$

- The parameter α_j is an intercept of the supply function for *j*-th firm, and β_j - slope of the supply function for player *j*.

FORMULATION OF THE SUPPLY FUNCTION MODEL

- A complete production cost is a quadratic function:

$$C_j(q_j) = 0.5 d_j q_j^2 + a_j q_j.$$

- a_j is an intercept of the marginal cost function for j-th firm and d_j - slope of the marginal cost function for j-th firm.
- The system demand curve is assumed to be:

$$D_t = N(t) - \gamma p_t.$$

- $N(t)$ is a load-time function, representing the load at time t if price were zero, and γ is a slope of the relationship between demand and price.

THE FORWARD CONTRACTS AS A SECURITY OF THE PLAYERS' PROFITS

- The profit for *j*-th firm is given by the following equation:

$$\pi_{jt} = p_t q_j(p_t) - C_j(q_j(p_t)), \quad t \in [t_0, U).$$

- At time, when the forward contract is accounted we obtain:

$$\pi_{jU} = p_U q_j(p_U) - C_j(q_j(p_U)) + F_{jT}^U (P_{jT}^U - \sum_{k=T}^{U} p_k).$$

- F_{jT}^U is a forward contract obligation, P_{jT}^U is a forward contract price.

THE EQUILIBRIUM POINT IN CASE WITH THE FORWARD CONTRACT

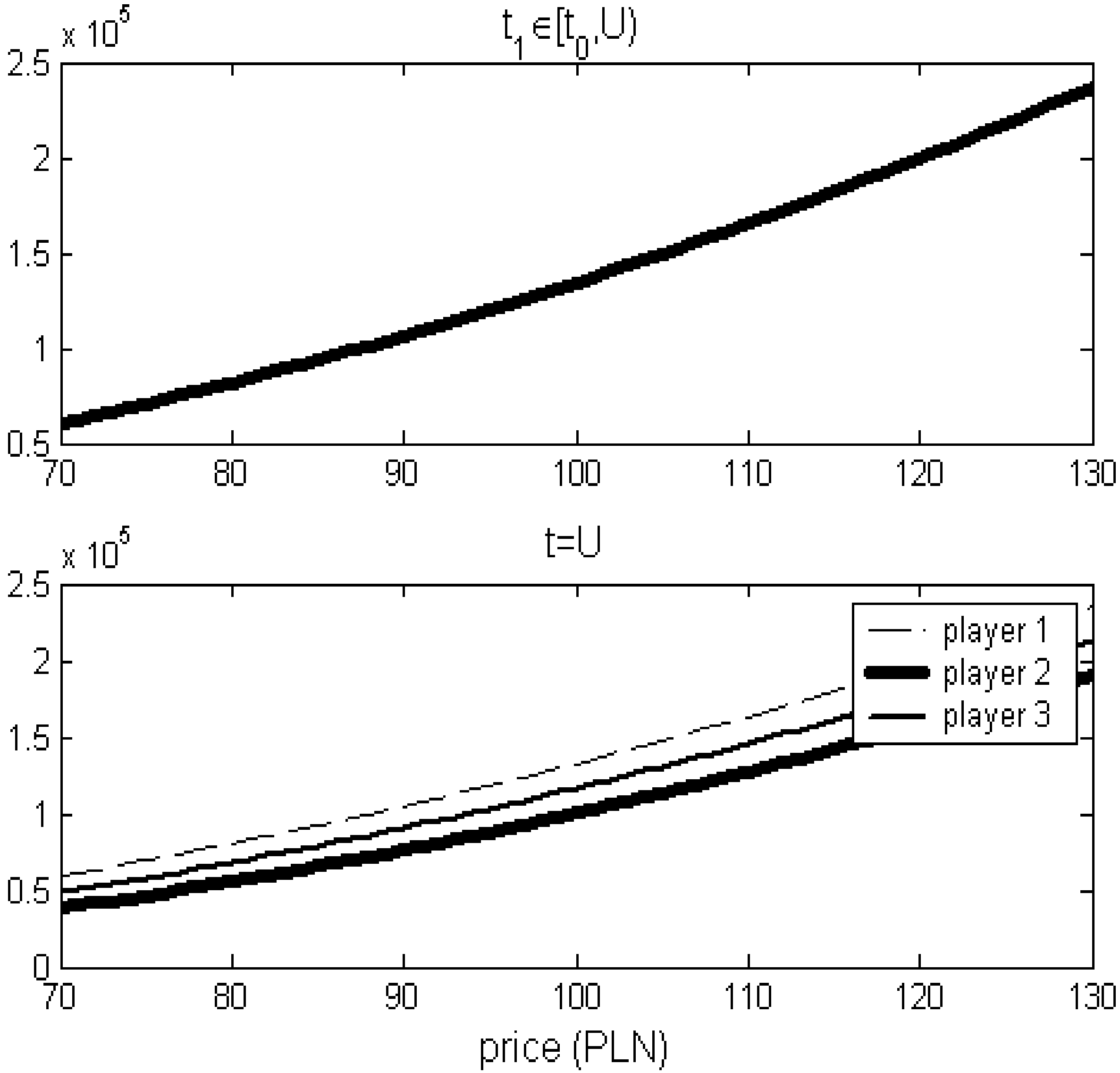
- For each $t \in [t_o, U)$ we obtain:

$$\beta_j^* = \frac{\gamma + \sum_{k \neq j} \beta_k^*}{d_j(\gamma + \sum_{k \neq j} \beta_k^*) + 1}, \qquad \alpha_j^* = a_j.$$

- At time $t=U$ we have

$$\beta_j^* = \frac{2}{d_j} = \frac{\gamma + \sum_{k \neq j} \beta_j^*}{d_j(\gamma + \sum_{k \neq j} \beta_j^*) + 1} \qquad \alpha_j^* = a_j - \left(\frac{1}{\beta_j^*} - d_j \right) F_{jT}^U = a_j + \frac{d_j F_{jT}^U}{2}.$$

EXAMPLE -CASE 3 PLAYERS WITH EQUAL PRODUCTION COSTS



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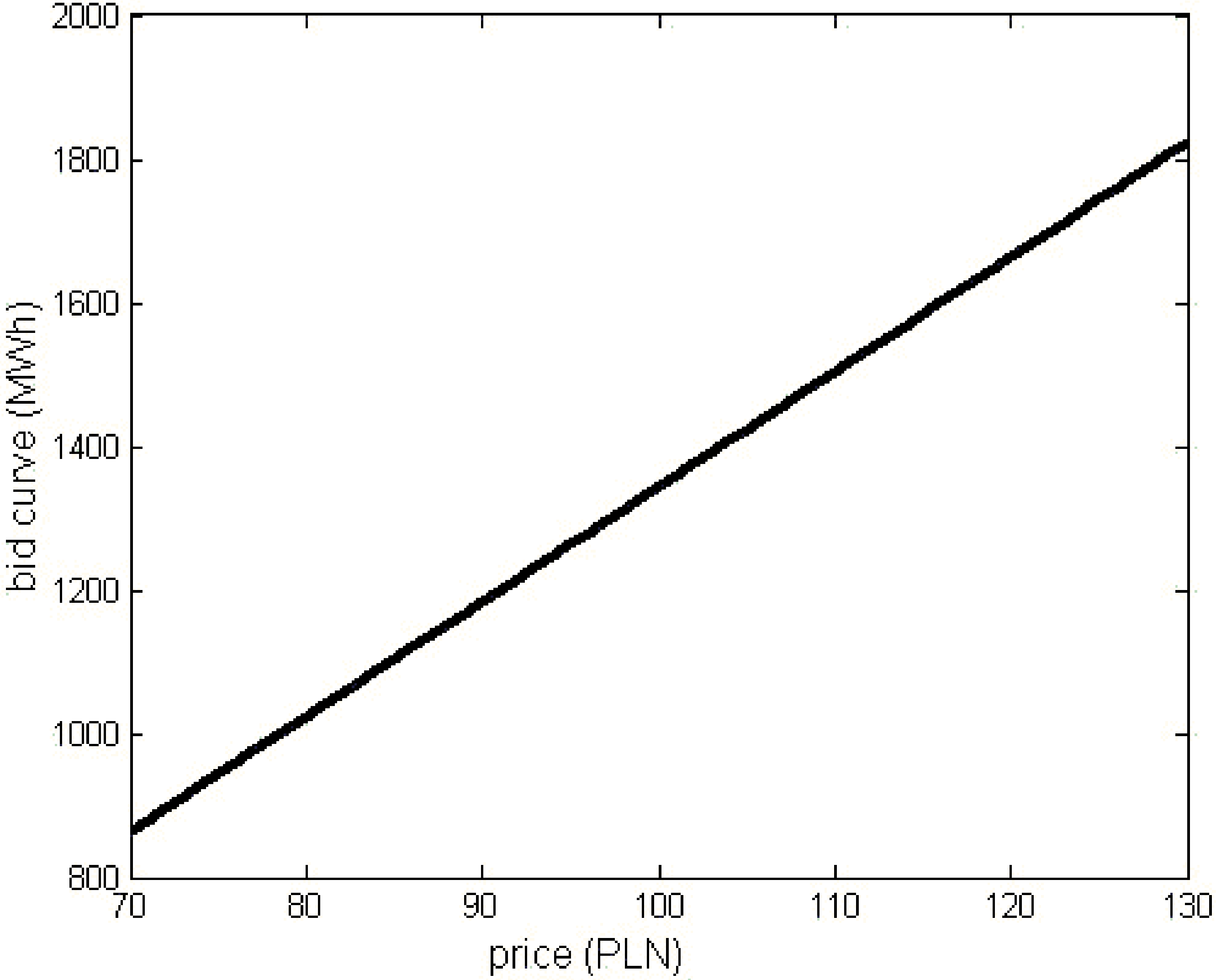

$$t \in [t_0, U)$$

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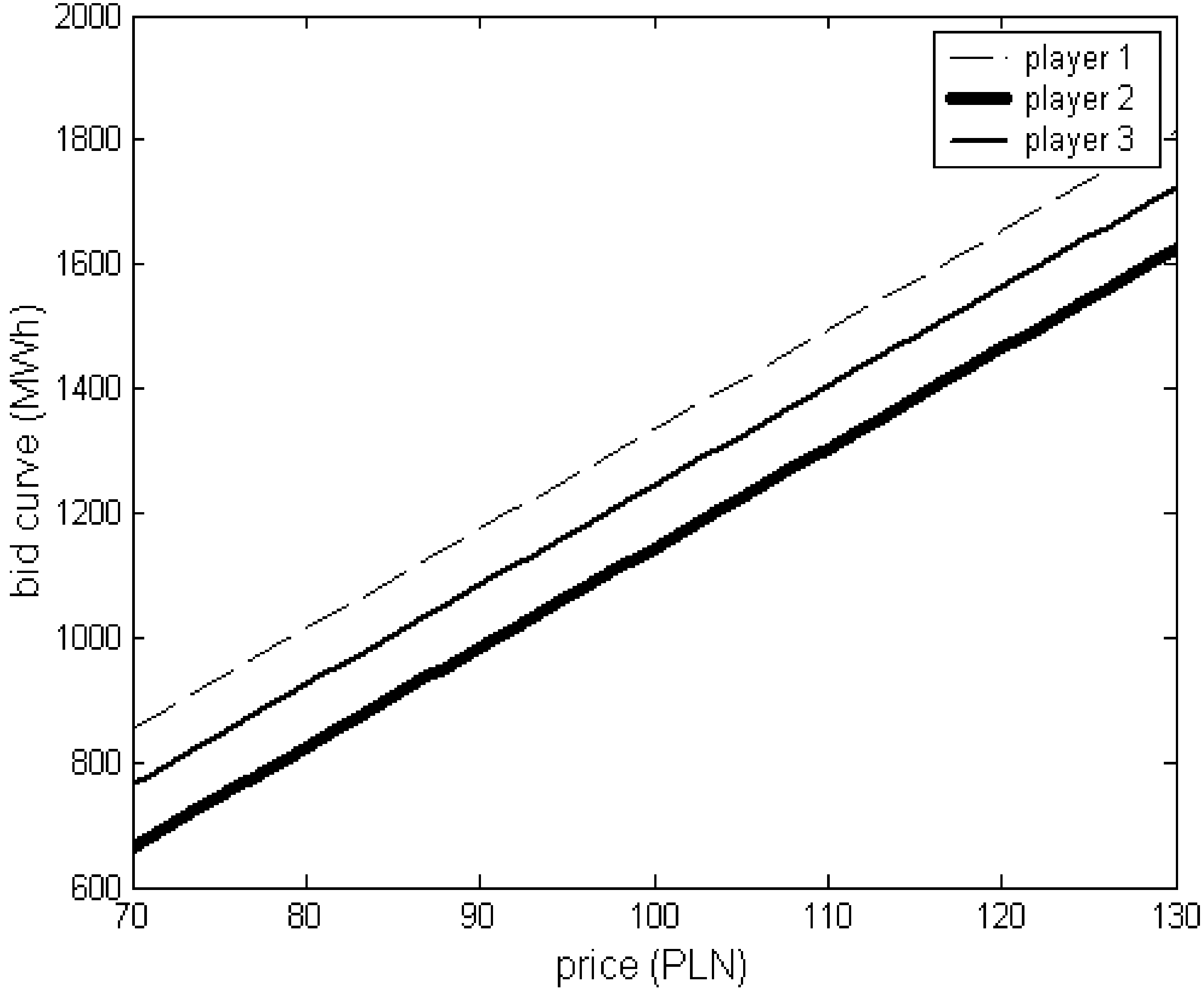
EXAMPLE -CASE 3 PLAYERS WITH EQUAL PRODUCTION COSTS



$t = U$

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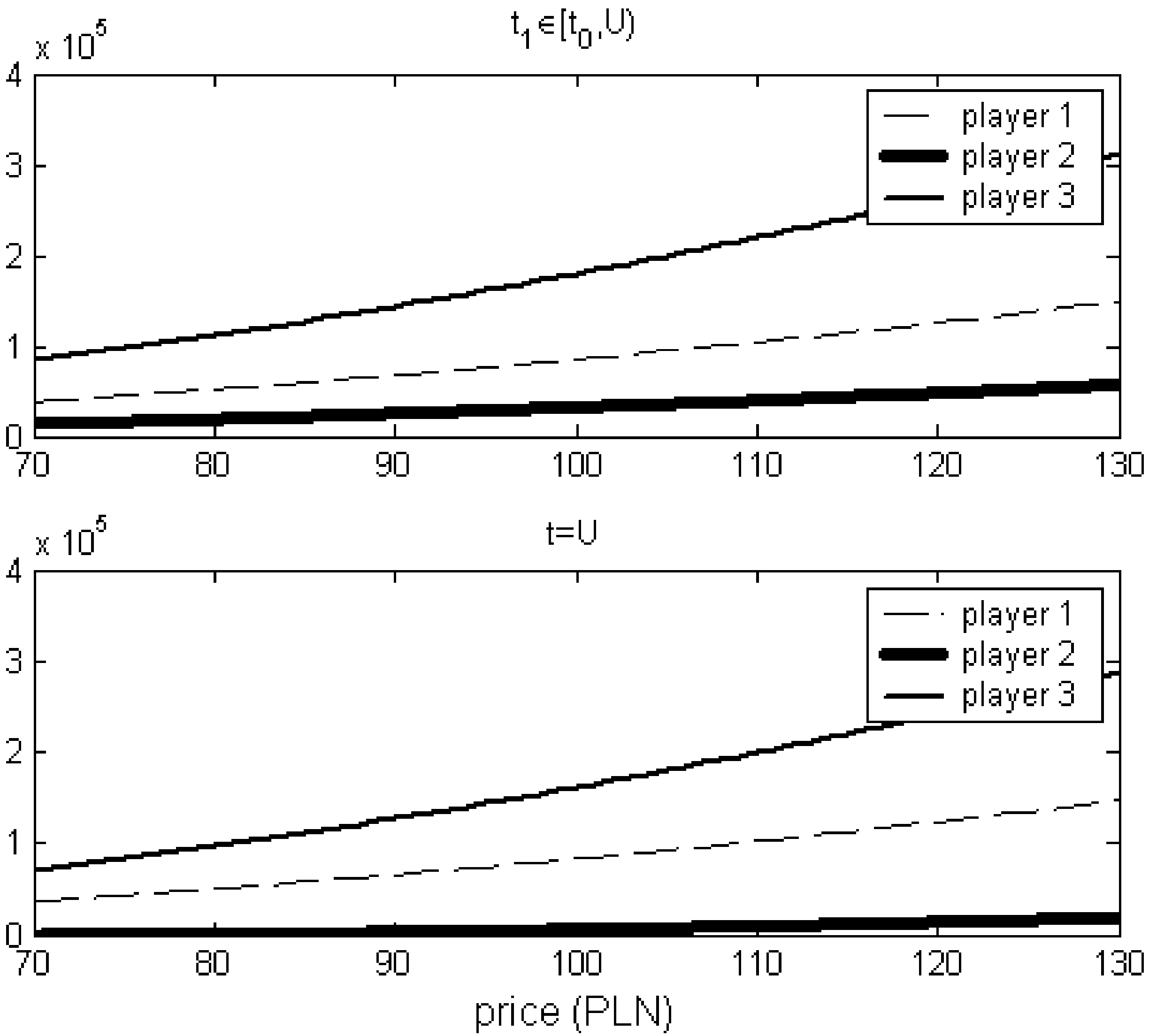
EXAMPLE -CASE 3 PLAYERS WITH DIFFERENT PRODUCTION COSTS



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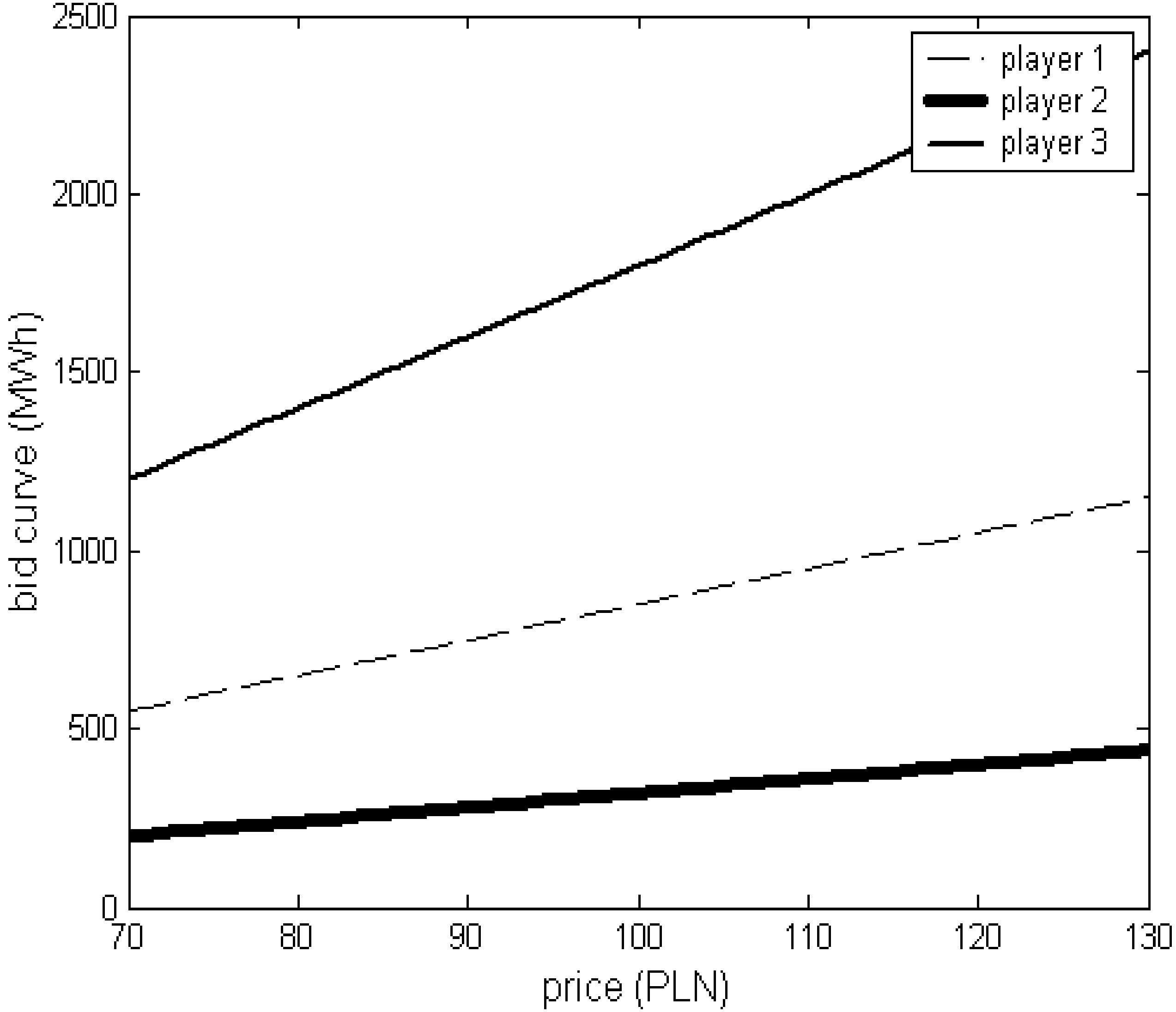
EXAMPLE -CASE 3 PLAYERS WITH DIFFERENT PRODUCTION COSTS



$$t \in [t_0, U)$$

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THE EUROPEAN CALL OPTIONS AS A SECURITY OF THE PLAYERS' PROFITS

- The profit for *j*-th firm is given by the following equation:

$$\pi_{jt} = p_t q_j(p_t) - C_j(q_j(p_t)), \qquad t \in [t_0, U).$$

- At time, when the option is accounted we obtain:

$$\pi_{jU} = p_U q_j(p_U) - C_j(q_j(p_U)) - A_{jT}^U (\tilde{P}_{jT}^U - \sum_{k=T}^{U} p_k)_+ .$$

- A_{jT}^U is an option obligation, $\tilde{P}_{jT}^U$ is an option price.

THE EQUILIBRIUM POINT IN CASE WITH THE EUROPEAN OPTION

- For each $t \in [t_o, U)$ we obtain:

$$\beta_j^* = \frac{\gamma + \sum_{k \neq j} \beta_k^*}{d_j(\gamma + \sum_{k \neq j} \beta_k^*) + 1}, \qquad \alpha_j^* = a_j.$$

- At time $t = U$ we have

$$\alpha_j^* = \begin{cases} a_j & if \quad p_U < \widetilde{P}_{jT}^U - \sum_{k=T}^{U} p_k \\ a_j - \frac{d_j A_{jT}^U}{2} & if \quad p_U > \widetilde{P}_{jT}^U - \sum_{k=T}^{U} p_k. \end{cases}$$

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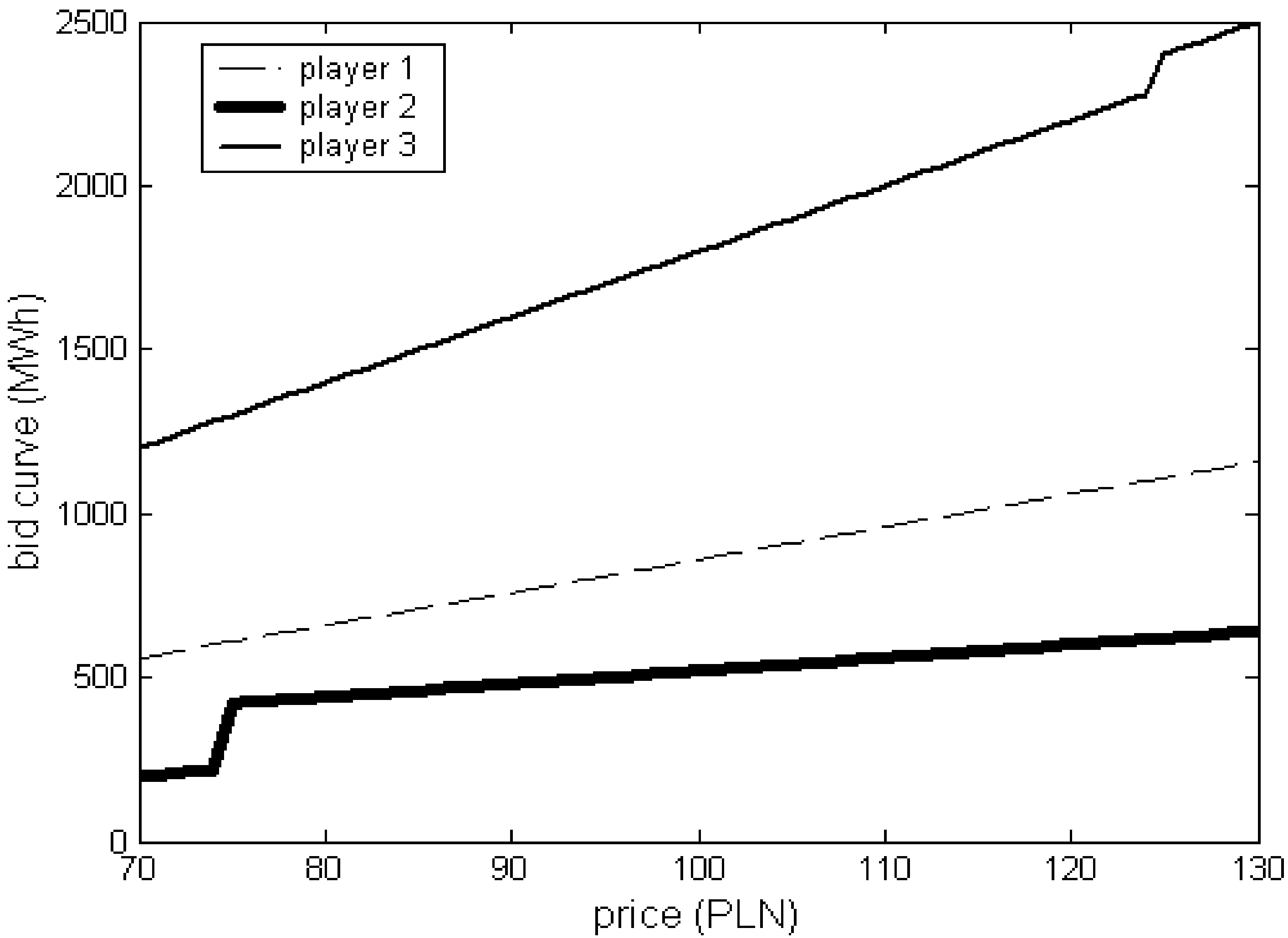

$t = U$

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[4] Niu H., Baldick R., Zhu G., Supply function equilibrium bidding strategies with fixed forward contracts. IEEE Transactions on Power Systems, Vol. 20, No. 4, November 2005.