Modeling Electricity Prices with Regime Switching Models

Michael Bierbrauer[1], Stefan Trück[1], and Rafał Weron[2]

[1] Chair of Statistics, Econometrics and Mathematical Finance, University of Karlsruhe, Karlsruhe, 76128, Germany
stefan@statistik.uni-karlsruhe.de
http://www.statistik.uni-karlsruhe.de/htm/mitarbeiter/trueck.php
[2] Hugo Steinhaus Center for Stochastic Methods, Wrocław University of Technology, 50-370 Wrocław, Poland
rweron@im.pwr.wroc.pl

Abstract. We address the issue of modeling spot electricity prices with regime switching models. After reviewing the stylized facts about power markets we propose and fit various models to spot prices from the Nordic power exchange. Afterwards we assess their performance by comparing simulated and market prices.

1 Electricity Spot Prices: Markets and Models

The deregulation of the power industry has given way to a global trend toward the commoditization of electric energy. Electricity has transformed from a primarily technical business, to one in which the product is treated in much the same way as any other commodity, with trading and risk management as key tools to run a successful business [2,12,15]. However, we have to bear in mind that electricity is a very unique commodity. It cannot be economically stored, demand of end users is largely weather dependent, and the reliability of the transmission grid is far from being perfect. This calls for adequate models of price dynamics capturing the main characteristics of spot electricity prices.

The spot electricity market is actually a day-ahead market. A classical spot market would not be possible, since the system operator needs advanced notice to verify that the schedule is feasible and lies within transmission constraints. The spot is an hourly (in some markets – a daily) contract with physical delivery. In our analysis we use spot prices from the Nordic power exchange (Nord Pool) covering the period January 1, 1997 – April 25, 2000. The system price is calculated as the equilibrium point for the aggregated supply and demand curves and for each of the 24 hours [14]. Due to limited space, in this paper we restrict the analysis to average daily prices. The averaged time series, however, retains the typical characteristics of electricity prices, including seasonality (on the annual and weekly level), mean reversion and jumps [20,21].

The seasonal character of electricity spot prices is a direct consequence of the fluctuations in demand. These mostly arise due to changing climate conditions,

M. Bubak et al. (Eds.): ICCS 2004, LNCS 3039, pp. 859–867, 2004.

like temperature and the number of daylight hours. In the analyzed period the annual cycle can be quite well approximated by a sinusoid with a linear trend [20,21]. The weekly periodicity is not sinusoidal, though, with peaks during the weekdays and troughs over the weekends. Spot electricity prices are also regarded as mean reverting – for time intervals ranging from a day to almost four years the Hurst exponent is significantly lower than 0.5 [18,19]. In addition to seasonality and mean reversion, spot electricity prices exhibit infrequent, but large jumps caused by extreme load fluctuations (due to severe weather conditions, generation outages, transmission failures, etc.). The spot price can increase tenfold during a single hour but the spikes are normally quite short-lived [2,12, 15,21].

Now, that we have discussed the properties of spot electricity prices we can turn to modeling issues. The starting point is the analysis of seasonal components. On the annual level this can be done through approximation by sinusoidal functions [15,21], fitting a piecewise constant function of a one year period [1, 13] or wavelet decomposition [18]. On the weekly (or daily) time scale, the seasonality is usually removed by subtracting an average week (or day) from the data. Once the seasonal components are removed we are left with the stochastic part of the process. In what follows we will analyze the logarithm d_t of the deseasonalized average daily spot prices D_t, see the bottom panel in Figure 1. For details on obtaining d_t from raw data see [20,21].

The stochastic part d_t can be modeled by a diffusion-type stochastic differential equation (SDE) of the form: $dX_t = \mu(X,t)dt + \sigma(X,t)dB_t$, which is the standard model for price processes of stochastic nature. Mean reversion is typically induced into the model by having a drift term $\mu(X,t)$ that is negative if the spot price is higher than the mean reversion level and positive if it is lower, like in the arithmetic Ornstein-Uhlenbeck process:

$$dX_t = (\alpha - \beta X_t)dt + \sigma dB_t = \beta(L - X_t)dt + \sigma dB_t, \tag{1}$$

where $\mu(X,t) = (\alpha - \beta X_t)$ is the drift, $\sigma(X,t) = \sigma$ is the volatility and dB_t are the increments of a standard Brownian motion. This is a one-factor model that reverts to the mean $L = \frac{\alpha}{\beta}$ with β being the magnitude of the speed of adjustment. The equilibrium level L can be also made time dependent to reflect the fact that electricity prices tend to revert to different levels over the year.

The second main feature of electricity spot prices, the "jumpy" character, calls for spot price modeling which is not continuous. One approach is to introduce to eqn. (1) a jump component $J_t dq_t$, where J_t is a random jump size and q_t is a Poisson variate [2,10]. After a spike the price is forced back to its normal level by the mean reversion mechanism or mean reversion coupled with downward jumps. Alternatively, a positive jump may be always followed by a negative jump of the same size to capture the rapid decline – especially on the daily level – of electricity prices after a spike [20,21].

Since spot prices after a jump tend to remain high for several time periods (hours, sometimes even days) there is also need for models that are able to capture this behavior. The so-called regime switching models offer such a possibility and be discussed in the next section.

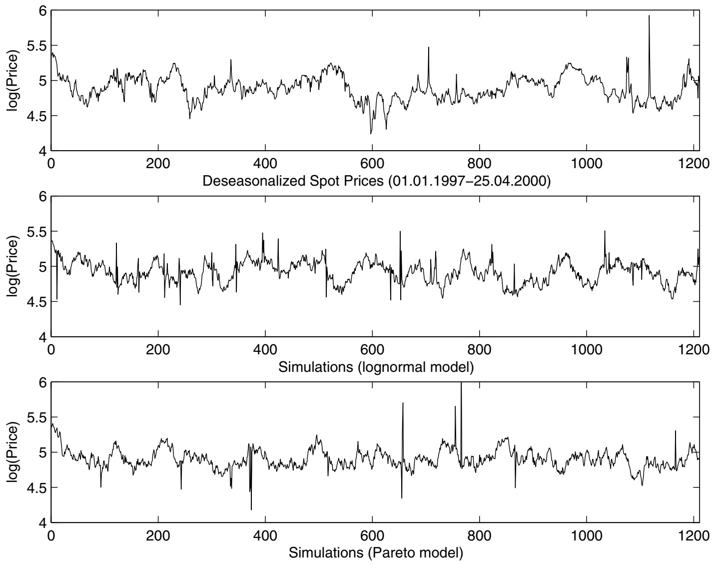

Fig. 1. The deseasonalized log-price process d_t for the time period 01.01.1997-25.04.2000 (*top panel*) and sample simulated price trajectories obtained from the two-regime model with normal (*middle panel*) and Pareto (*bottom panel*) distributions for the spike regime

2 Regime Switching Models

The price behavior of spot electricity prices can be modeled by dividing the time series into separate phases or regimes with different underlying processes. A jump in electricity prices can then be considered as a change to another regime [4,8,9]. The switching mechanism is typically assumed to be governed by a random variable that follows a Markov chain with different possible states. Thus, we have an unobservable variable in the time series that switches between a certain number of states which themselves are driven by independent stochastic processes [5,6,7,16]. Additionally we have a probability law that governs the transition from one state to another.

2.1 Two-Regime Models

To introduce the idea of regime switching models we start with the simplest model with two possible states. The two-regime model distinguishes between a base regime ($R_t = 1$) and a spike regime ($R_t = 2$), i.e. the spot price is supposed to display either mean reverting or jump behavior at each point of time. The price processes $Y_{t,1}$ and $Y_{t,2}$ that are linked to each of the two regimes are assumed to be independent of each other. The variable R_t that determines the current state is a random variable that follows a Markov chain with two possible states,

$R_t = \{1, 2\}$. The transition matrix $\mathbf{P}$ contains the probabilities p_{ij} of switching from regime i at time t to regime j at time $t+1$:

$$\mathbf{P} = (p_{ij}) = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{pmatrix}. \tag{2}$$

The current state R_t of a Markov chain depends on the past only through the most recent value R_{t-1}. Thus $P\{R_t = j | R_{t-1} = i\} = p_{ij}$. The probability of being in state j at time $t+m$ starting from state i at time t is given by:

$$\begin{pmatrix} P(R_{t+m} = 1 | R_t = i) \\ P(R_{t+m} = 2 | R_t = i) \end{pmatrix} = (\mathbf{P}')^m \cdot e_i, \tag{3}$$

where $\mathbf{P}'$ denotes the transpose of $\mathbf{P}$ and e_i denotes the ith column of the 2×2 identity matrix.

There are various possibilities for choosing the stochastic processes for the base and the peak regime. However, considering the typical behavior of electricity spot prices described in the previous section, we let the base regime ($R_t = 1$) be governed by a mean-reverting process, eg. given by eqn. (1). In the spike regime ($R_t = 2$) it may be interesting to try different types of distributions for the process $Y_{t,2}$. The Gaussian [9] and lognormal [20] laws were suggested in the literature so far. In the latter case the deseasonalized log-price process d_t is defined by $dY_{t,1} = (c_1 - \beta_1 Y_{t,1})dt + \sigma_1 dB_t$ in the base regime and $\log(Y_{t,2}) \sim \mathrm{N}(c_2, \sigma_2^2)$ in the spike regime. The parameter set $\theta = \{c_1, \beta_1, \sigma_1, c_2, \sigma_2, p_{11}, p_{22}\}$ can be estimated using the so-called EM algorithm [3].

2.2 Alternative Regime Switching Models

Clearly the variety of regime switching models is due to both the possibility of choosing the number of regimes (2, 3, etc.) and different stochastic process for the price in each regime. Especially for the spike regime it may be interesting to choose alternative distributions. Since spikes happen very rarely but usually are of great magnitude the use of heavy-tailed distributions should be considered. We therefore suggest the use of the Pareto distribution (see e.g. [11]) for the spike regime. Also the process that switches between a certain number of states should be chosen in accordance with the typical behavior of spot electricity prices.

Huisman and Mahieu [8] propose a regime switching model with three possible regimes. The idea behind their specification differs significantly from the previous two-state models. They identify three possible regimes: (i) the regime $R_t = 1$ modeling the "normal" electricity price dynamics, (ii) an initial jump regime $R_t = 2$ for a sudden increase or decrease in price, and (iii) a regime $R_t = 3$ that describes how prices move back to the normal regime after the initial jump has occurred. This definition implies that the initial jump regime is immediately followed by the reversing regime and then moves back to the base regime. Thus we get a 3×3 transition matrix with only four non-zero values: p_{11}, $p_{12} = 1 - p_{11}$, $p_{23} = 1$, and $p_{31} = 1$.

Furthermore, Huisman and Mahieu [8] suggest to model the base and reversing jump regimes by a mean reverting process and the initial jump regime by Brownian motion (i.e. a process with increments given by a Gaussian variate). However, we do not see the need for modeling the reversing jump regime with a mean reverting process. The process automatically leaves this regime after one time period and it seems that a Gaussian or a lognormal random variable will do the job as well. The direction of the initial jump is not specified; it can be either an upward or a downward jump. However, we restrict the model so that the reversal jump, on average, is opposite to the initial jump. Hence, our three-regime model is defined by $dY_{t,1} = (c_1 - \beta_1 Y_{t,1})dt + \sigma_1 dB_t$ in the base regime, $\log(Y_{t,2}) \sim \mathrm{N}(c_2, \sigma_2^2)$ in the initial jump regime and $\log(Y_{t,3}) \sim \mathrm{N}(-c_2, \sigma_2^2)$ in the reversing jump regime. In contrast to the two-regime models, the three-regime model does not allow for consecutive spikes (or remaining at a different price level for two or more periods after a jump). In the next section we will compare estimation and simulation results of different regime switching models.

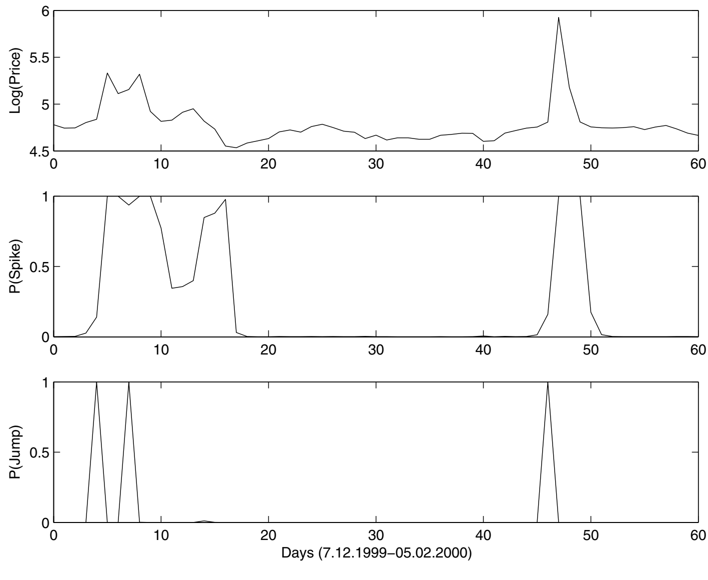

Fig. 2. The deseasonalized log-spot price d_t since December 7, 1999 until February 5, 2000 (*top panel*) together with the probability of being in the spike regime for the estimated two-regime model with lognormal spikes (*middle panel*) and of being in the jump regime for the estimated three-regime model (*bottom panel*)

Table 1. Estimation results for the two-regime model and the deseasonalized log-price d_t for the period January 1, 1997 – April 25, 2000. $E(Y_{t,i})$ is the level of mean reversion for the base regime ($i = 1$) and the expected value of the spike regime ($i = 2$), p_{ii} is the probability of remaining in the same regime in the next time step, and $P(R = i)$ is the unconditional probability of being in regime i

Two-regime model with Gaussian spikes

	β_i	c_i	σ_i^2	$E(Y_{t,i})$	p_{ii}	$P(R=i)$
Base regime ($i=1$)	0.0426	0.2078	0.0018	4.8807	0.9800	0.9484
Spike regime ($i=2$)	—	—	0.0610	4.9704	0.6337	0.0512

Two-regime model with lognormal spikes

	β_i	c_i	σ_i^2	$E(Y_{t,i})$	p_{ii}	$P(R=i)$
Base regime ($i=1$)	0.0426	0.2078	0.0018	4.8807	0.9800	0.9484
Spike regime ($i=2$)	—	1.6018	0.0600	4.9678	0.6325	0.0516

Two-regime model with Pareto spikes

	β_i	c_i	σ_i^2	$E(Y_{t,i})$	p_{ii}	$P(R=i)$
Base regime ($i=1$)	0.0459	0.2241	0.0020	4.8822	0.9860	0.9699
Spike regime ($i=2$)	—	—	0.8294	4.9980	0.5497	0.0301

Three-regime model

	c_i	mu_i	σ_i^2	$E(Y_{t,i})$	p_{ii}	$P(R=i)$
Base regime ($i=1$)	0.2328	—	0.0024	4.8731	0.9924	0.9851
Init. Jump regime ($i=2$)	—	0.0839	0.0697	-	0	0.0075
Rev. Jump regime ($i=3$)	—	-0.0839	0.0697	-	0	0.0075

3 Empirical Analysis

In this section we analyze and model the logarithm d_t of the deseasonalized average daily spot prices from the Nord Pool power exchange since January 1, 1997 until April 25, 2000. For details on obtaining d_t from raw data see [20,21]. As we can see in Figure 1, the data exhibits several extreme events that can be considered as spikes. While most spikes only last for one day there are periods where the prices exhibit three or more extreme events in a row, a behavior that could be considered as consecutive spikes, see the top panel in Figure 2. This is the motivation for fitting the two-regime models with the base regime dynamics given by $dY_{t,1} = (c_1 - \beta_1 Y_{t,1})dt + \sigma_1 dB_t$ and the dynamics in the spike regime following a normal, a lognormal or a Pareto distribution, see Section 2.1. For comparison, we also fit the three-regime model described in the previous section.

The estimation results are summarized in Table 1. In all models, the probability of remaining in the base regime is quite high: in the two-regime model we get $p_{11} = 0.9800$ for the normal and lognormal model specifications and $p_{11} = 0.9860$ for the Pareto specification. For the three-regime model we get an extremely low probability of leaving the base regime $p_{11} = 0.9924$. However, while in the three-regime model the price level immediately returns to the mean-reversion process after a jump, estimating the two-regime model we find $p_{22} = 0.6325$ for the normal, $p_{22} = 0.6337$ for the lognormal, and $p_{22} = 0.5497$ for the Pareto model. Thus, in all three models the probability of staying in the spike regime is quite high, see also Figure 2. The data points with a high probability of being in the

Table 2. Measures of the goodness-of-fit (Mean Squared Error – MSE, Mean Absolute Error – MAE, Loglikelihood – LogL) for the estimated regime switching models. Performance of the models is also assessed by comparing the number of spikes, the return distributions quantiles ($q_{0.99}$ and $q_{0.995}$), and the extreme events

	MSE	MAE	LogL	spikes	$q_{0.99}$	$q_{0.995}$	max	min
Real Data	–	–	–	9.00	0.1628	0.2235	1.1167	-0.7469
2-regime (normal)	0.0047	0.0403	1890.28	17.26	0.3310	0.4523	0.7580	-0.8038
2-regime (lognormal)	0.0047	0.0403	1890.67	18.05	0.3353	0.4648	0.7937	-0.7875
2-regime (Pareto)	0.0047	0.0402	1866.11	33.32	0.5410	0.7851	2.1688	-2.2602
3-regime model	0.0048	0.0398	1854.56	13.72	0.3087	0.4144	0.7347	-0.6883

jump regime ($P\{R_t = 2\} > 0.5$) tend to be grouped in blocks in the two-regime models. Due to model specifications, in the three-regime model the probability of remaining in the second regime is zero.

Considering the unconditional probabilities we find that there is a 5.16%, 5.12% and 3.01% probability of being in the spike regime for the Gaussian, lognormal and Pareto two-regime models, respectively. This value is substantially larger than the probability of a jump in the three-regime model which is approximately equal to $P(R = 2) = P(R = 3) = 0.75\%$. Surprisingly, the normal and lognormal distributions produce almost identical results. A closer inspection of the parameter estimates uncovers the mystery – with such a choice of parameter values the lognormal distribution very much resembles the Gaussian law. However, using a heavy-tailed distribution, like the Pareto law, gives lower probabilities for being and remaining in the spike regime and a clearly higher variance.

Simulated price trajectories were used to check for similarity with real prices and stability of results. Reestimating the models with simulated data led to only slightly biased estimates for the parameters. Sample trajectories for the two-regime model with lognormal and Pareto spikes can be found in Figure 1. The trajectories of both models show strong similarity to real price data. We also checked the simulation results considering spikes as the most particular feature of electricity spot prices, see Table 2. Defining a spike as a change in the log-prices that is greater than 0.3 – either in positive or negative direction – we find that the regime switching models produce more spikes than there could be observed in real data. Especially in the two-regime model the number of spikes in simulations is about twice the number that was observed in the considered period. While the number of extreme events are overestimated in all models (see the values of $q_{0.99}$ and $q_{0.995}$ in Table 2), the magnitude of the largest spike in either direction is underestimated in the normal and lognormal models and overestimated by the Pareto distribution. This may suggest the use of alternative heavy-tailed distributions, e.g. a truncated Pareto or a stable distribution with parameter $\alpha > 1$ [17] for the spike regime.

4 Conclusions

This paper addresses the issue of modeling spot electricity prices. For the deseasonalized log-prices d_t we propose different regime switching models, which exhibit mean reversion and jump behavior.

We find that the models produce estimates for transition probabilities that can be interpreted according to market behavior. Simulated trajectories show high similarity with real price data. However, we find that the number of price spikes or extreme events produced by simulations of the estimated models is higher than what could be observed in real price data. This is especially true for the two-regime models where consecutive spikes have a higher probability than in the three-regime model.

References

1. K. Bhanot, Behavior of power prices: Implications for the valuation and hedging of financial contracts. The Journal of Risk 2 (2000) 43-62
2. L. Clewlow, C. Strickland, Energy Derivatives – Pricing and Risk Management, Lacima Publications, London, 2000
3. A. Dempster, N. Laird, D. Rubin, Maximum likelihood from incomplete data via the EM algorithm, J. Royal Statist. Soc. 39 (1977) 1-38
4. R. Ethier, T. Mount, Estimating the volatility of spot prices in restructured electricity markets and the implications for option values, Cornell University Working Paper 12/1998
5. S. Goldfeld, R. Quandt, A Markov Model for Switching Regressions, J. Econometrics 1 (1973) 3-16
6. J.D. Hamilton, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, Econometrica 57 (1989) 357-384
7. J.D. Hamilton, Time Series Analysis, Princeton University Press, 1994.
8. R. Huisman, R. Mahieu, Regime jumps in electricity prices, Energy Economics 25 (2003) 425-434
9. R. Huisman, C. de Jong, Option pricing for power prices with spikes, Energy Power Risk Management 7.11 (2003) 12-16
10. B. Johnson, G. Barz, Selecting stochastic processes for modelling electricity prices, in: Risk Publications, Energy Modelling and the Management of Uncertainty, Risk Books (1999) 3-21
11. N. Johnson, S. Kotz and Narayanaswamy Balakrishnan, Continuous Univariate Distributions, Wiley, New York, 1995
12. V. Kaminski (ed.) Managing Energy Price Risk, Risk Books, London, 1999
13. J.J. Lucia, E.S. Schwartz, Electricity prices and power derivatives: Evidence from the Nordic Power Exchange, Rev. Derivatives Research 5 (2002) 5-50
14. Nord Pool, Nord Pool Annual Report, Nord Pool ASA, 2002
15. D. Pilipovic, Energy Risk: Valuing and Managing Energy Derivatives, McGraw-Hill, New York, 1998
16. R. Quandt, The Estimation of the Parameters of a linear Regression System Obeying two Separate Regimes, J. Amer. Statist. Assoc. 55 (1958) 873-880
17. C. Mugele, S.T. Rachev and S. Trück, Analysis of Different Energy Markets with the alpha-stable distribution, University of Karlsruhe Working Paper 12/2003

18. I. Simonsen, Measuring anti-correlations in the Nordic electricity spot market by wavelets, Physica A 322 (2003) 597-606
19. R. Weron, B. Przybyłowicz, Hurst analysis of electricity price dynamics, Physica A 283 (2000) 462-468
20. R. Weron, M. Bierbrauer, S. Trück, Modeling electricity prices: jump diffusion and regime switching, Physica A (2004) to appear
21. R. Weron, I. Simonsen, P. Wilman, Modeling highly volatile and seasonal markets: evidence from the Nord Pool electricity market, in H. Takayasu (ed.), The Application of Econophysics, Springer, Tokyo, 2004, 182-191