

A SIMPLE MODEL OF PRICE FORMATION

K. SZNAJD-WERON

*Institute of Theoretical Physics, University of Wrocław
pl. Maza Borna 9, 50-204 Wrocław, Poland
kweron@ift.uni.wroc.pl*

R. WERON

*Hugo Steinhaus Center, Wrocław University of Technology
Wyspińskiego 27, 50-370 Wrocław, Poland
rweron@im.pwr.wroc.pl*

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A simple Ising spin model, which can describe the mechanism of price formation in financial markets is proposed. In contrast to other agent-based models, the influence does not flow inward from the surrounding neighbors to the center site, but spreads outward from the center to the neighbors. The model thus describes the spread of opinions among traders. It is shown via standard Monte Carlo simulations that very simple rules lead to dynamics that duplicate those of asset prices.

Keywords: Ising model; agent-based model; econophysics; price dynamics.

1. Introduction

The Ising spin system is one of the most frequently used models of statistical mechanics. Its simplicity (binary variables) makes it appealing to researchers from other branches of science including biology,¹ sociology² and economy.³⁻⁸ It is rather obvious that Ising-type models cannot explain origins of very complicated phenomena observed in complex systems. However, it is believed that these kind of models can describe some universal behavior.

Recently, an Ising spin model, which can describe the mechanism of making a decision in a closed community was proposed.⁹ In spite of simple rules the model exhibited complicated dynamics in one and more¹⁰ dimensions. In contrast to usual majority rules,¹¹ in this model the influence was spreading outward from the center. This idea seemed appealing and we adapted it to model financial markets. We introduced new dynamic rules describing the behavior of two types of market players: trend followers and fundamentalists. The obtained results were astonishing — the properties of simulated price trajectories duplicated those of analyzed historic

data sets. Three simple rules led to a fat-tailed distribution of returns, long-term dependence in volatility and no dependence in returns themselves.

We strongly believe that this simple and parameter free model is a good first approximation of a number of real financial markets. But before we introduce our model we review some of the stylized facts about price formation in the financial markets.

2. Stylized Facts

Adequate analysis of financial data relies on an explicit definition of the variables under study. Among others these include the price and the change of price. In our studies the price x_t is the daily closing price for a given asset. The change of price r_t at time t is defined as $r_t = \log x_{t+1} - \log x_t$. In fact, this is the change of the logarithmic price and is often referred to as return. The change of price, rather than the price itself, is the variable of interest for traders (and researchers as well).

Fat-tailed distribution of returns. — The variety of opinions about the distributions of asset returns and their generating processes is wide. Some authors claim the distributions to be closed to Paretian stable,¹² some to generalized hyperbolic,^{13,14} and some reject any single distribution.^{15,16}

Instead of looking at the central part of the distribution, an alternative way is to look at the tails. Most types of distributions can be classified into three categories: 1° thin-tailed — for which all moments exist and whose density function decays exponentially in the tails; 2° fat-tailed — whose density function decays in a power-law fashion; 3° bounded — which have no tails.

Virtually all quantitative analysts suggest that asset returns fall into the second category. If we plot returns against time we can notice many more outlying (away from the mean) observations than for white noise. This phenomenon can be seen even better on normal probability plots, where the cumulative distribution function (CDF) is drawn on the scale of the cumulative Gaussian distribution function. Normal distributions have the form of a straight line in this representation, which is approximately the case for the distribution of weekly or monthly returns. However, distributions of daily and higher-frequency returns are distinctly fat-tailed.¹⁷ This can be easily seen in the top panels of Fig. 1, where daily returns of the DJIA index for the period Jan. 2nd, 1990–Dec. 30th, 1999, are presented.

Clustering and dependence. — Despite the wishes of many researchers asset returns cannot be modeled adequately by series of iid (independent and identically distributed) realizations of a random variable described by a certain fat-tailed distribution. This is caused by the fact that financial time series depend on the evolution of a large number of strongly interacting systems and belong to the class of complex evolving systems. As a result, if we plot returns against time we can observe the nonstationarity (heteroscedasticity) of the process in the form of clusters, i.e., periods during which the volatility (measured by standard deviation or the equivalent l^1 norm¹⁸) of the process is much higher than usual, see the top-left panel of

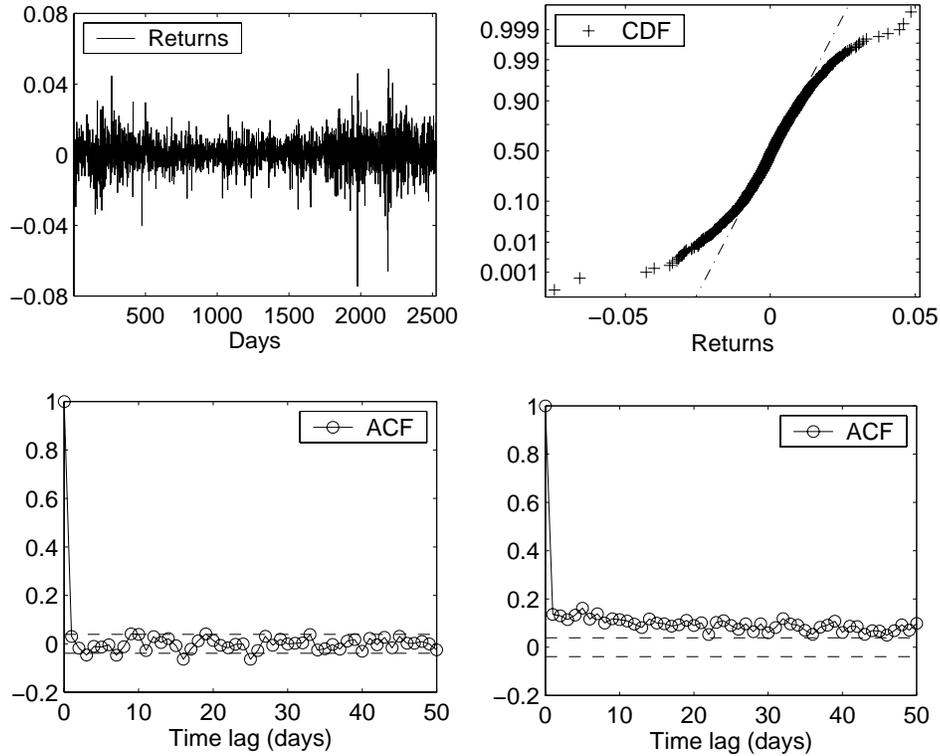


Fig. 1. Daily returns of the Dow Jones Industrial Average index during the last decade (top left), normal probability plot of DJIA returns (top right), lagged autocorrelation function of DJIA daily returns (bottom left), and lagged autocorrelation function of absolute value of DJIA daily returns (bottom right). Dashed horizontal lines represent the 95% confidence interval of a Gaussian random walk.

Fig. 1. Thus it is natural to expect dependence in asset returns. Fortunately, there are many methods to quantify dependence. The direct method consists in plotting the autocorrelation function:

$$\mathbf{acf}(r, k) = \frac{\sum_{t=k+1}^N (r_t - \bar{r})(r_{t-k} - \bar{r})}{\sum_{t=1}^N (r_t - \bar{r})^2},$$

where N is the sample length and $\bar{r} = (1/N) \sum_{t=1}^N r_t$, for different time lags k . For most financial data autocorrelation of returns dies out (or more precisely: falls into the confidence interval of Gaussian random walk) after at most a few days and long-term autocorrelations are found only for squared or absolute value of returns,^{18–20} see bottom panels of Fig. 1. Recall that for Brownian motion — the classical model of price fluctuations^{19,21} — autocorrelations of r_t , r_t^2 and $|r_t|$ are not significant for lags greater or equal to one.

Another way to examine the dependence structure is the power spectrum analysis, also known as the “frequency domain analysis”. One of the most often used

techniques was proposed by Geweke and Porter-Hudak (GPH)²² and is based on observations of the slope of the spectral density function of a fractionally integrated series around the angular frequency $\omega = 0$. A simple linear regression of the periodogram I_n (a sample analogue of the spectral density) at low Fourier frequencies

$$\omega_k : \log\{I_n(\omega_k)\} = a - \hat{d} \log\left\{4 \sin^2\left(\frac{\omega_k}{2}\right)\right\} + \epsilon_k$$

yields the differencing parameter $d = H - 0.5$ through the relation $d = \hat{d}$. The GPH estimate of the Hurst exponent H has well known asymptotic properties and allows for construction of confidence intervals.^{22,23}

Yet another method is the Hurst R/S analysis^{24,25} or its “younger sister” — the Detrended Fluctuation Analysis (DFA).²⁶ Both methods are based on a similar algorithm, which begins with dividing the time series (of returns) into boxes of equal length and normalizing the data in each box by subtracting the sample mean (R/S) or a linear trend (DFA). Next some sort of volatility statistics is calculated (rescaled range or mean square fluctuation, respectively) and plotted against box size on a double-logarithmic paper. Linear regression yields the Hurst exponent H , whose value can lie in one of the three regimes: 1° $H > 0.5$ — persistent time series (strict long memory), 2° $H = 0.5$ — random walk or a short-memory process, 3° $H < 0.5$ — anti-persistent (or mean-reverting) time series. Unfortunately, no asymptotic distribution theory has been derived for the R/S and DFA statistics so far. However, it is possible to estimate confidence intervals based on Monte Carlo simulations.²³

3. The Model

Recently a simple model for opinion evolution in a closed community was proposed.^{9,10} In this model (called USDF) the community is represented by a horizontal chain of Ising spins, which are either up or down. A pair of parallel neighbors forces its two neighbors to have the same orientation (in random sequential updating), while for an antiparallel pair, the left neighbor takes the orientation of the right part of the pair, and the right neighbor follows the left part of the pair.

In contrast to usual majority rules,¹¹ in the USDF model the influence does not flow inward from the surrounding neighbors to the center site, but spreads outward from the center to the neighbors. The model thus describes the spread of opinions. The dynamic rules lead to three steady states: two ferromagnetic (all spins up or all spins down) and one antiferromagnetic (an up-spin is followed by a down-spin, which is again followed by an up-spin, etc.).

In this paper we modify the model to simulate price formation in a financial market. The spins are interpreted as market participants’ attitude. An up-spin ($S_i = 1$) represents a trader who is bullish and places buy orders, whereas a down-spin ($S_i = -1$) represents a trader who is bearish and places sell orders. In our model the first dynamic rule of the USDF model remains unchanged, i.e., *if $S_i S_{i+1} = 1$ then S_{i-1} and S_{i+2} take the direction of the pair $(i, i+1)$* . This can be justified by the

fact that a lot of market participants are trend followers and place their orders on the basis of a local guru's opinion. However, the second dynamic rule of the USDF model has to be changed to incorporate the fact that the absence of a local guru (two neighboring spins are in different directions) causes market participants to act randomly rather than make the opposite decision to his neighbor: *if $S_i S_{i+1} = -1$ then S_{i-1} and S_{i+2} take one of the two directions at random.*

Such a model has two stable states (both ferromagnetic), which is not very realistic for a financial market. Fortunately, trend followers are not the only participants of the market.²⁷ There are also fundamentalists — players that know much more about the system and have a strategy (or perhaps we should call them “rationalists”). To make things simple, in our model we introduce one fundamentalist, somewhat similar to Bornholdt's model.^{28,29} He knows exactly what is the current difference between demand and supply in the whole system. If supply is greater than demand he places buy orders, if lower — sell orders.

It is not clear *a priori* how to define the price in a market. The only obvious requirement is that the price should go up when there is more demand than supply, and vice versa. For simplicity, we define the price x_t in our model as the normalized difference between demand and supply (magnetization): $x_t = (1/N) \sum_{i=1}^N S_i(t)$, where N is the system size. Note that $x_t \in [-1, 1]$, so $|x_t|$ can be treated as probability. Now we can formulate the third rule of our model: *the fundamentalist will buy (i.e., take value 1) at time t with probability $|x_t|$ if $x_t < 0$ and sell (i.e., take value -1) with probability $|x_t|$ if $x_t > 0$.*

The third rule means that if the system will be closed to the stable state “all up”, the fundamentalist will place sell orders with probability close to one (in the limiting state exactly with probability one) and start to reverse the system. So the price will start to fall. On the contrary, when r will be closed to -1 , the fundamentalist will place buy orders (take the value 1) and the price will start to grow. This means that ferromagnetic states will not be stable states anymore.

4. Results

To investigate our model we perform a standard Monte Carlo simulation with random updating. We consider a chain of N Ising spins with free boundary conditions. We were usually taking $N = 1000$, but we have done simulations for $N = 10\,000$ as well. We start from a totally random initial state, i.e., to each site of the chain we assign an arrow with a randomly chosen direction: up or down (Ising spin).

In our simulations each Monte Carlo step (MCS) represents one trading hour; eight steps constitute one trading day. We typically simulate 20 000 MCSs, which corresponds to 2500 trading days or roughly 10 years. We chose such a period of time, because in this paper we compare the empirical results with historical data sets (two FX rates and two stock indices) of about the same size: 2245 daily quotations of the dollar–mark (USD/DEM) exchange rate for the period Aug. 9th, 1990–Aug. 20th, 1999 (see Fig. 2 and Table 1), 2809 daily quotations of the yen-dollar

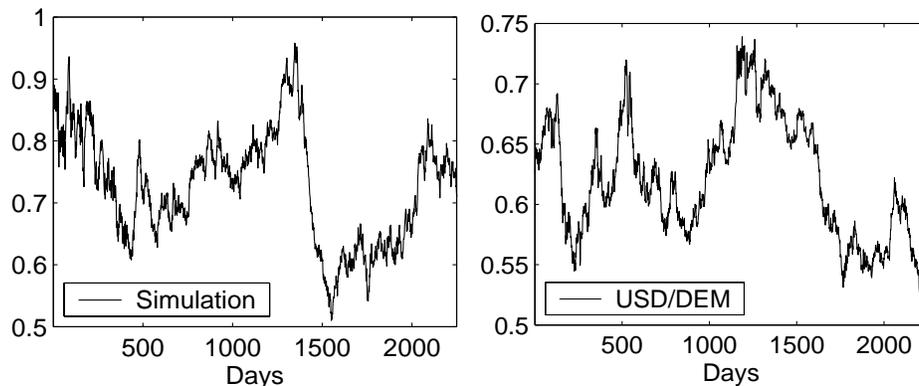


Fig. 2. A typical path of the simulated price process x_t and the USD/DEM exchange rate, respectively.

Table 1. Estimates of the Hurst exponent H for simulated and market data.

Data	Method		
	R/S-AL	DFA	GPH
	Returns		
Simulation	0.5270	0.4666	0.3653
USD/DEM	0.5127	0.5115	0.6154
JPY/USD	0.5353	0.5303	0.5790
DJIA	0.4585	0.4195**	0.3560
WIG20	0.5030	0.4981	0.4606
	Absolute value of returns		
Simulation	0.8940***	0.9335***	0.8931***
USD/DEM	0.7751***	0.8406***	0.8761***
JPY/USD	0.8576***	0.9529***	0.9287***
DJIA	0.7838***	0.9080***	0.8357***
WIG20	0.9103***	0.9494***	0.8262***

*, ** and *** denote significance at the (two-sided) 90%, 95% and 99% level, respectively. For the R/S-AL and DFA statistics inference is based on empirical Monte Carlo results of Weron,²³ whereas for the GPH statistics — on asymptotic distribution of the estimate of H .²²

(JPY/USD) exchange rate for the period Jan. 2nd, 1990–Feb. 28th, 2001 (see Table 1), 2527 daily quotations of the Dow Jones Industrial Average (DJIA) index for the period Jan. 2nd, 1990–Dec. 30th, 1999 (see Fig. 1), and 1561 daily quotations of the WIG20 Warsaw Stock Exchange index (based on 20 blue chip stocks from the Polish capital market) for the period Jan. 2nd, 1995–Mar. 30th, 2001 (see Table 1).

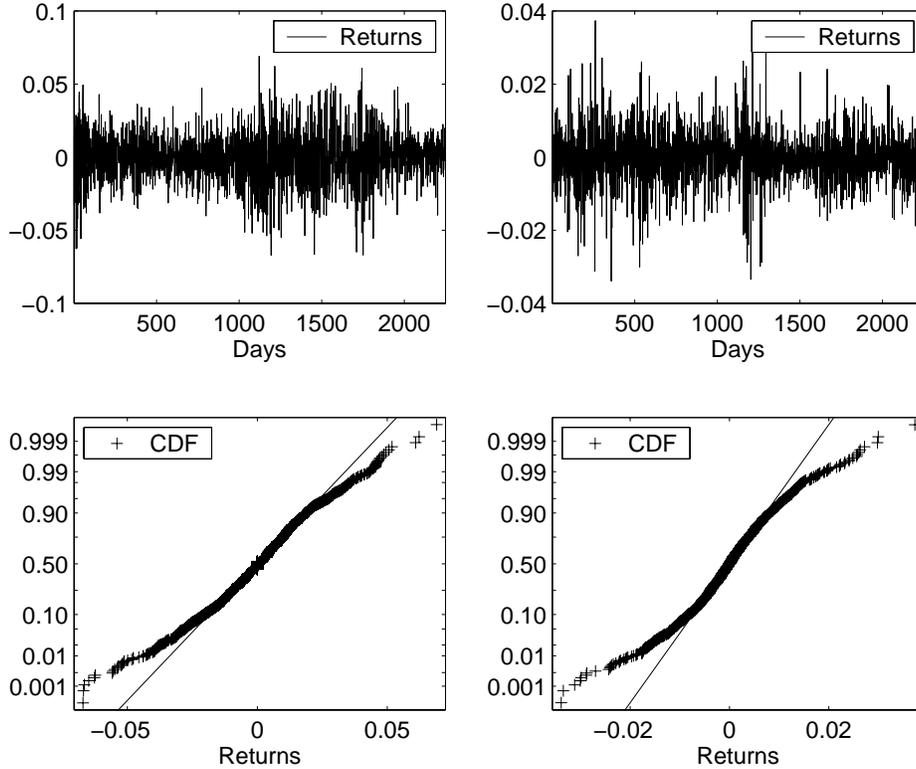


Fig. 3. Returns of the simulated price process x_t and daily returns of the USD/DEM exchange rate during the last decade, respectively (top panels). Normal probability plots of r_t and USD/DEM returns, respectively, clearly showing the fat tails of price returns distributions (bottom panels).

The returns r_t are obtained from the simulated price curve x_t (see Fig. 2) after it is shifted (incremented by one) to make it positive. Alternatively we could have defined the up-spin to be equal to two and the down-spin $-$ to zero. However, this would have made the calculations more difficult and the description of the model less appealing.

In Fig. 3, we present daily returns and normal probability plots for the simulated (left panels) and USD/DEM exchange rate (right panels) time series. Without prior knowledge as to the magnitude of historical returns it is impossible to judge which process is real and which is simulated. The same is true for the simulated and the DJIA returns of Fig. 1.

In Fig. 4, we present the lagged autocorrelation study for the simulated (left panels) and USD/DEM exchange rate (right panels) time series. Again the properties of the simulated price process duplicate those of historical data. The lagged autocorrelation of returns falls into the confidence interval of Gaussian random walk

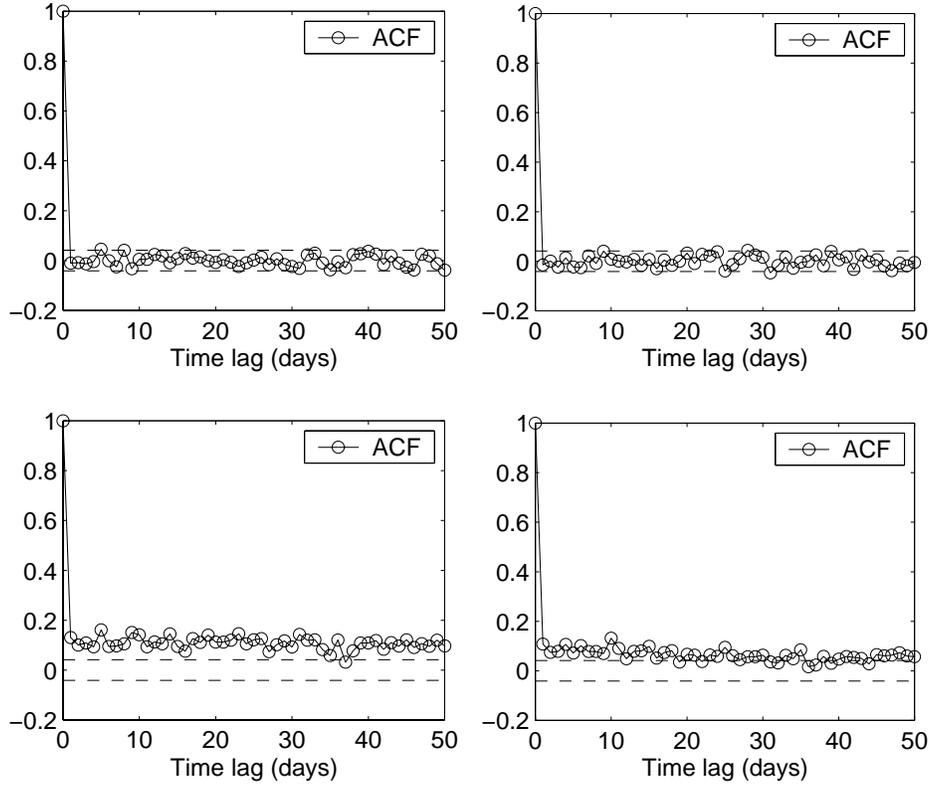


Fig. 4. Lagged autocorrelation functions of r_t and USD/DEM returns, respectively (top panels). Lagged autocorrelation functions of absolute value for the same data sets (bottom panels). Dashed horizontal lines represent the 95% confidence interval of a Gaussian random walk.

immediately, like for the dollar–mark exchange rate. The same plot for the DJIA returns shows a bit more dependence.

This can be seen also in Table 1, where values and significance of the R/S, DFA and GPH statistics for the simulated and four historical data sets are presented. In all but one case (DFA for DJIA) the Hurst exponents of daily returns are insignificantly different from those of white noise. On the other hand, the Hurst exponents of the absolute value of daily returns are persistent in all cases, with the results being significant even at the two-sided 99% level. Moreover, the estimates of H from the simulation are indistinguishable from those of real market data.

The presented empirical analysis clearly shows that three simple rules of our model lead to a fat-tailed distribution of returns, long-term dependence in volatility and no dependence in returns themselves as observed for market data. Thus we may conclude that this simple model is a good first approximation of a number of real financial markets.

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