Stochastic modeling of time series with application to local damage detection in rotating machinery

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Abstract. Raw vibration signals measured on a machine housing in industrial conditions are complex and can be modeled as an additive mixture of several processes (with different statistical properties) related to normal operation of machine, damage related to one (or more) of its part, some noise, etc. In case of local damage in rotating machines, contribution of informative process related to damage is hidden in the raw signal so its detection is difficult. In this paper we propose to use the statistical modeling of vibration time series to identify these components. Building a model of raw signal may be ineffective. It is proposed to decompose the signal into set of narrowband sub-signals using a time-frequency representation. Next, it is proposed to model each sub-signal in the given frequency range and classify all sub-signals using their statistical properties. We have used several parameters (called selectors because they will be used for selection of sub-signals from the time-frequency map for further processing) for analysis of the sub-signals. They are based on the statistical analysis and can be useful for example in testing Gaussianity of the data set (vibration time series from machine in good condition is close to Gaussian, damaged not). Results of such modeling will be used in the sub-signals classification procedure and in the defects detection as well. We illustrate effectiveness of the novel technique using real data from heavy machinery system.

1. Introduction

Local damage in rotating machinery is one of the critical damage occurring in practice, so researchers show great interest in this issue. Different approaches can be found in the literature (see review works [1,2]). From the signal processing perspective, the most known approaches try to understand properties of the signal in the time or frequency domain using advanced signal processing techniques for pre-processing and analyzing the signal. Just a few authors considered machine's response as time series that can be modeled [3,4] in a stochastic sense. In the case of bearings, several works discussed application of statistical signal processing to build a model of time series using autoregressive models [4,5,6,7,8,9]. In the case of gearboxes, it is much more difficult because vibrations of a gearbox are much more complex than bearing's, moreover, "locally faulty" gearbox produces non-stationary time series consisted of deterministic and stochastic components [4,10,11,12] that should be separated first in order to use stochastic modeling [13]. In practical situations, even for bearings diagnostics the problem of deterministic contamination is present in the raw signal. Separation problem, or in other words, extraction of informative part of the signal is one of the most important issues for local damage detection. The last decade has provided many powerful approaches exploiting wavelets [14,15,16] (and other time-frequency techniques [17,18]), cyclostationarity of the signal [2,4,19], adaptive/optimal filters [20,21,22,23,24] and so on. One of family of techniques is related to so-called optimal frequency band, i.e. containing information with the best signal to noise ratio. Ideas provided by Antoni (spectral kurtosis, kurtogram, spectral coherence map, etc., see [2] for review) and their adaptations done by different authors ([17,18,24,25]) confirmed practically that for local damage detection it is much easier if the signal is properly enhanced, i.e. pre-filtered. Obviously the most critical question is how to find such a filter that is able to extract information about damage.

To do this, in this paper we propose to use novel technique based on the statistical properties of examined time series. The idea of novel approach is to use statistical modeling for narrowband sub-signals and use results to classify the sub-signals. Each sub-signal is time series (for a given narrow frequency range) that arises after (for example time-frequency) decomposition. It is well known that impulsive (in the time domain) excitation related to damage will produce wideband disturbance of the spectrum. It is expected that for a set of frequency bands (containing information about damage) properties of sub-signals will be different than for other (non-informative). In some sense the idea is similar to the spectral kurtosis, however different statistical estimators (called selectors) are used.

The analyzed statistical properties are expressed as the distance between empirical distribution and the base one that in the considered case is Gaussian. The Gaussian distribution (called also normal) is considered the most prominent probabilistic distribution in statistics. There are many reasons for this. The first one is related to the fact that the normal distribution arises from the central limit theorem, which states that under mild conditions, the mean of a large number of random variables independently drawn from the same distribution is approximately normally distributed, irrespective of the form of the original distribution. Secondly, the normal distribution is very tractable analytically, that is, a large number of results involving this distribution can be derived in explicit form, [26]. For these reasons, the normal distribution is commonly encountered in practice and is used as a simple model for complex phenomena. In vibro-diagnostics the Gaussian distribution was examined in many papers. It is said that machine in good condition (gearbox, bearing) produces vibration signal with approximately Gaussian distribution.

The most popular measure of Gaussianity is kurtosis (K=3 indicates normal distribution, higher values inform about deviation from Gaussian distribution). The proposed measures expressed as the distance between empirical and Gaussian distribution are alternative for mentioned method based on the analysis of kurtosis and can provide more effective results. They allow us to recognize whether the examined sub-signal is close to the Gaussian or not. The proposed idea provides to the simple test for local damage detection in rotating machinery.

The rest of the paper is organized as follows. In Section 2 we present the layout of the procedure that consists of decomposition of examined raw vibration signal into sub-signals via time-frequency spectrogram. Next, the statistical tests, called selectors, are presented. Section 3 contains results of analysis with application to the real vibration signal. The last section provides final conclusions and remarks on further work.

2. Novel procedure for signal processing

In this paper we propose the novel procedure for signal analyzing. The raw vibration time series is complex , so it is very difficult to describe it using simple fault-related models. Therefore before the further analysis the signal should be decomposed into a set of simpler sub-signals. In the next step, we analyze the relation between time-series related to each sub-signal. The signal with significant correlation cannot be modeled using the tools presented below, so there is a need to check these properties. One of the measures that are useful in this case is the autocorrelation function.

Sub-signal containing cyclic impulsive contribution with suitable signal to noise ratio is expected to exhibit significant non-Gaussian property of time series (in opposite to time series representing machine in good condition). In order to test it several approaches, called selectors, are proposed. Moreover we also propose to compare the mentioned selectors to the classical measure, namely kurtosis.

2.1. Decomposition

As it was said, modeling of a complex raw signal is very difficult due to non-informative contributions from different sources. In order to make the analysis more effective, we decompose the raw vibration signal into sub-signals. It can be done in many ways, here it is done by the Short-Time Fourier Transform (STFT) [27] that for observations $X_1, X_2, \ldots X_n$ is defined as:

$$STFT(t, f) = \sum_{k=1}^{N} X_k W_{k-t} e^{-jfk} \tag{1}$$

The raw signal is being segmented, windowed and the absolute value of the Fourier Transform for each window is calculated. Finally, two dimensional (time-frequency) map describing instantaneous frequency structure of the signal is estimated. If adjacent windows overlap, it results in dependency of the output data, i.e. subsequent values of time series for a fixed frequency may be dependent. To prevent this, we do not use overlapping here. Remark: It is well known that in a case of spectrogram there is a time-frequency resolution problem. It is obvious that for better accuracy of informative band estimation fine resolution in frequency is required (it means long window). From the other hand, to provide statistically reasonable estimators, a proper number of segments (it constitute single sample in sub-signal) is needed. This issue becomes more critical if overlapping is forbidden. To maintain this problem, we use relatively narrow windows, which provide sufficient frequency resolution.

2.2. Testing of data dependency for a single sub-signal

It is worth mentioning that statistical methods we use can be applied only for independent samples therefore before the further analysis we should confirm the examined sub-signal constitutes a sample of independent observations. One of the methods that are useful there is the sample autocorrelation function (ACF) which is an empirical equivalence of the autocorrelation function. For independent sample the ACF should be close to zero (i.e. it should be in the confidence interval for given confidence level).

In case of nonzero ACF such sub-signal will not be considered (will be treated as non-informative). More precisely, we set the value of the selector to 0 for sub-signals if their autocorrelation function exceeds the 95% confidence interval in significantly more than 5% of lags.

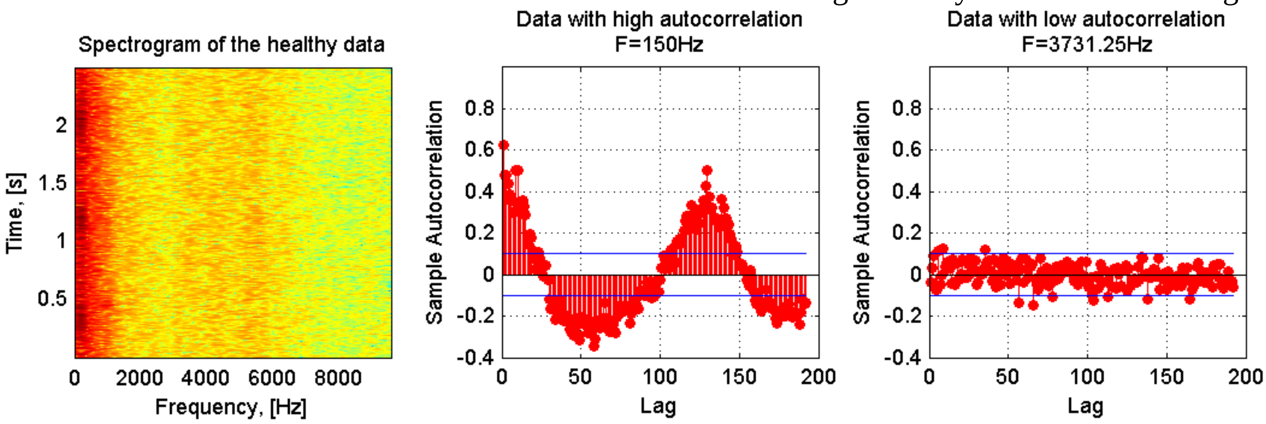

Fig. 1 Time-frequency representation of raw time series for the healthy bearing (left panel) and ACF for data with high (center panel) and low (right panel) sample autocorrelation.

2.3. Selectors

In this section we describe statistics, called selectors, which we use to analyze and classify the sub-signals. The idea of classification is based on measuring the distance between empirical distribution of underlying sub-signal and the base distribution. In the considered case we use the Gaussian distribution as a base. More precisely, we examine statistical properties of sub-signals represented by the distance between observed distribution of the data and the Gaussian one.

Information of a frequency band is evaluated on the basis of its selector's value. In a case of visual tests some parameters have been proposed to make the procedure automatic. As a result, we obtain the whole frequency band divided into classes of similar statistical properties.

The mentioned statistics (selectors) are connected with well-known statistical tests for normality.

The Kolmogorov-Smirnov statistic (K-S). In statistics, the Kolmogorov–Smirnov test is a nonparametric test for equality of continuous, one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution (one-sample K–S test), or to compare two samples (two-sample K–S test). The K-S statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between empirical distribution functions of two samples. The null distribution of this statistic is calculated under the hypothesis that both samples are drawn from the same distribution (in the two-sample case) or that the sample is drawn from the reference distribution (in the one-sample case). The two-sample K-S test is one of the most useful and general nonparametric methods for comparing two samples, as it is sensitive to differences in both location and shape of the empirical cumulative distribution functions of two samples. The K-S statistic calculated for the sample $X_1, X_2, ..., X_n$ is defined as follows [28]:

$$D = \sup_x |F_n(x) - F(x)|, \qquad (2)$$

where $F_n(x)$ is the empirical distribution function that for random sample $X_1, X_2, ..., X_n$ takes the following form:

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} 1_{\{X_i \le x\}} \qquad (3)$$

In the above definition 1_A denotes the indicator of the set A. Moreover in the definition of the Kolmogorov-Smirnov statistic $F(x)$ is the theoretical cumulative distribution function that in our case of measuring distance between empirical and Gaussian distribution is the cumulative distribution function of a normal distribution:

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx, \qquad (4)$$

for parameters $\mu \in R, \sigma > 0$.

On the basis of the above statistic we can measure the distance between observed and Gaussian behavior. When a defects do not appear we can suspect the distance between normal and observed distribution should be smaller than in case when the defects appear.

The Anderson-Darling test (A-D). The A-D statistic used in this test belongs to the Cramer-von Mises family of statistics which incorporates the idea of the quadratic norm. The Cramer-von Mises statistic is defined by:

$$Q = n\int_{-\infty}^{\infty} \{F_n(x) - F(x)\}^2 \phi(x) dx, \qquad (5)$$

where $\phi(x)$ is a suitable function which gives weights to the squared difference $\{F_n(x)-F(x)\}^2$. When $\phi(x)=[F(x)(1-F(x))]^{-1}$ then the above definition yields the Anderson-Darling statistic, [29].

The tests derived from the A-D statistics is called Anderson-Darling test. It is well known that the K-S test exhibits a poor sensitivity to deviations from the hypothetical distribution that occurs in the tails, whereas the A-D test is considered as one of the most powerful when the fitted distribution departs from the base distribution in this area. In general, tests belonging to this family are used not only to recognize the difference between observed distribution and the Gaussian one, but also in many cases of other distributions, [30,31].

The quantile-quantile plot (QQplot). Except of statistical tests with explicit hypothesis, there are some visual tests to compare two distributions, e.g. the theoretical distribution and the sample one. One of the most famous examples is the quantile-quantile plot (QQplot), [32]. Plot of the theoretical distribution quantiles versus the underlying ones can be useful to recognize goodness-of-fit. Straight line on the QQplot means that compared distributions have the same shape. Straight line with equal scales on the axes means equal distributions. If there is no straight line, one can compare, for example, tail heaviness of both distributions. In most of numerical packages (MATLAB, R) there is an additional straight line plotted to make analyses easier. This line connects two points: first and third quartiles of both distributions. To make this test numerical, we propose to measure the horizontal distance between QQplot markers and the additional straight line. For this test, we propose to measure the mean and maximum distances. The exemplary QQplot of a healthy signal (for a given frequency) is presented in Fig. 2 (left panel) while for the sub-signal with defect - on the right panel of Fig. 2. We observe the healthy signal is closer to Gaussian distribution than the unhealthy one. The horizontal distance between straight line and markers is significantly higher on the right panel. It is worth mentioning that left tails of both data sets do not follow Gaussian distribution, but this behavior is manifested in the same way in both cases, so it does not affect mean nor maximum horizontal distance.

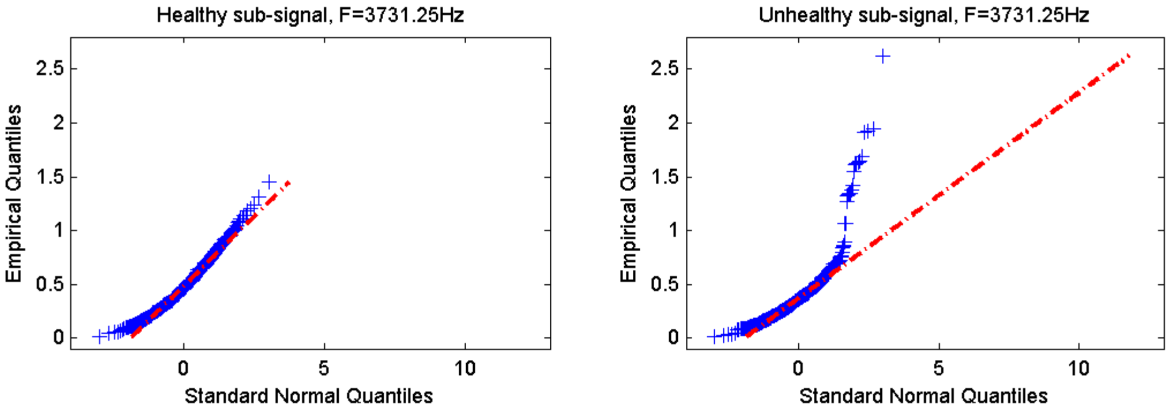

Fig. 2 QQplot of the healthy (left panel) and unhealthy (right panel) sub-signal compared to the normal distribution. The red line represents the reference quantile line for Gaussian distribution.

3. Applications to real data

The proposed procedure has been applied to vibration time series acquired from a machine (belt conveyor pulley's bearings) operating in a mining company (for a broader description of the machine see [8]). Two cases were considered: bearings in good condition and bearings in unhealthy condition (localized damage of outer ring). Duration of the signal was 2.5 s, sampling frequency fs=19200 Hz. The problem was to detect damage in presence of strong contamination from a gearbox located nearby [8]).

Results of our procedure are presented graphically as values of selectors versus frequency for the damaged and undamaged machine signals. As it can be easily seen for good condition machine data (plotted in blue), values of the selectors are similar for each frequency, while for unhealthy machine data (plotted in red) one can observe some frequency ranges with much higher values of the selectors. It means that for these frequencies the distance between empirical distribution and Gaussian one is greater than for the good condition machine data. It indicates the presence of damage. In order to compare the results, values of selectors are normalized by largest value occurring. One can see that similar ranges can be distinguished by the K-S, A-D and QQplot based selectors. Our results have been compared to well-known and widely used kurtosis (see Fig. 3, top right panel). One can conclude that A-D based selector seems to be the best one here, because the normalized distance between selectors for the healthy and unhealthy machine data seems to be the highest. What is more, A-D statistic values for the healthy signal is the least scattered. This makes setting a threshold distinguishing healthy and damaged machine easier. Normalized mean horizontal QQplot distance indicate similar dispersion, but the difference between healthy and unhealthy signal is smaller. Further analyzes of more suitable distributions than Gaussian must be performed to minimize the distance between healthy signal's empirical distribution and the base one.

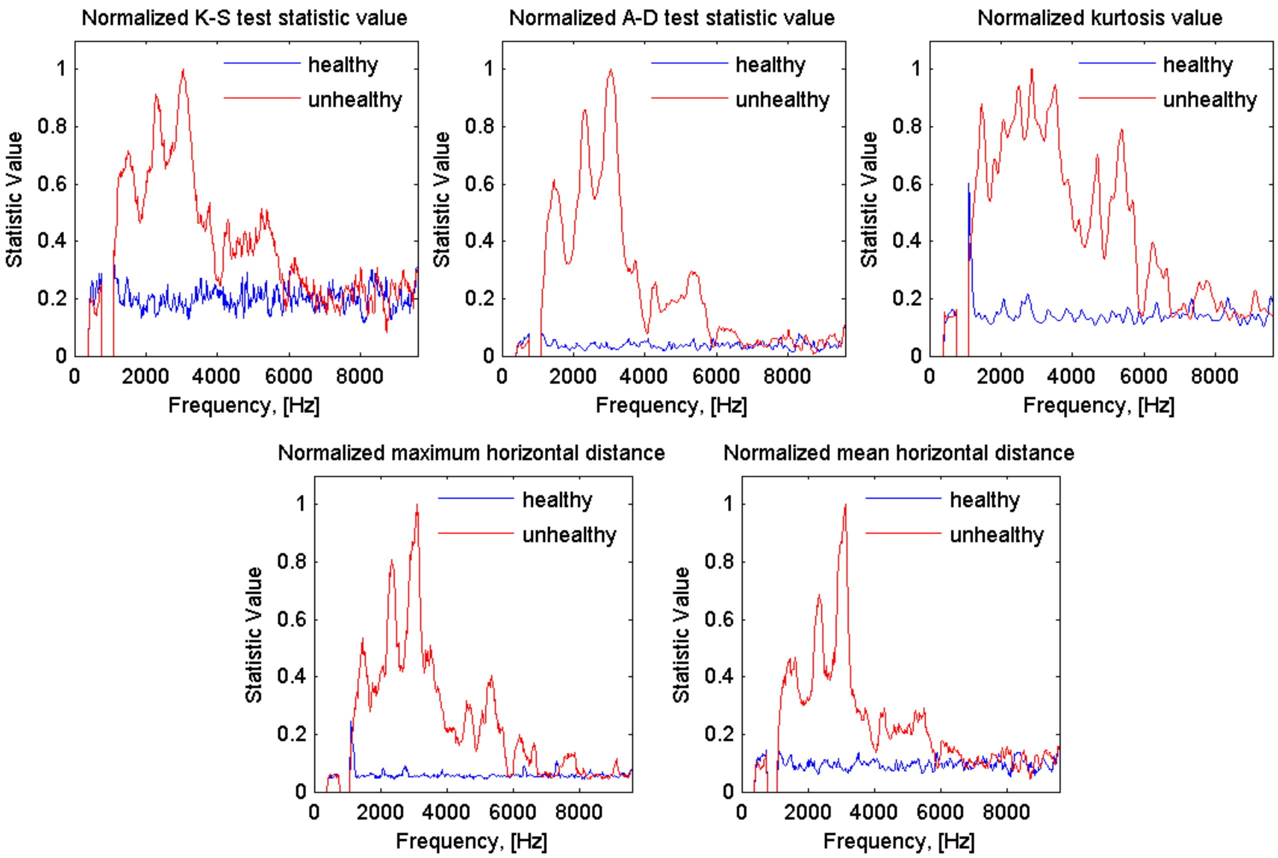

Fig. 3 Normalized values of analyzed selectors for healthy and unhealthy signals: K-S test statistic value (top left panel), A-D test statistic value (top center panel), kurtosis value (top right panel), maximum horizontal QQplot distance (bottom left panel) and mean horizontal QQplot distance (bottom right panel).

Summary

In this paper a novel approach for local damage detection is proposed. The methodology is based on statistical properties analysis of examined signals. It should be highlighted that this method has been developed for complex industrial signals. We propose several selectors that can be useful in the damage detection. They are based on the distance between examined empirical distribution and the well-known Gaussian one. We confirmed that for a real data set the presented procedure clearly distinguishes the healthy signal from unhealthy one. Our statistical analysis are preceded by decomposition of the underlying signal and examination of the relation between time-series. Our idea extends the methodology based on the kurtosis approach and gives more effective results.

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